Nov-2016

Seat Number

* 0 5 7 8 *

MATHEMATICS PAPER - I : MTH - 241

Complex Analysis (24115)

P. Pages : 2

Time : Two Hours **Max. Marks : 40**

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicate full marks.

1. Attempt **any eight** of the following. **8**

a) Find the amplitude of $\sqrt{3}+i$.

b) Find the sum of a complex number z = x + iy and its conjugate $\bar{z}$.

c) Evaluate $\lim_{z\to 1-i}[x+i(2x+y)]$.

d) Find the value of z at which $f(z)=\frac{z+2}{z^2-3z+2}$ is discontinuous in complex plane.

e) State the series for sin z.

f) State Cauchy integral formula for f' (a)

g) Find the poles of $f(z)=\frac{z+i}{(z^2+1)z}$.

h) Define harmonic function.

i) if $z=e^{i\theta}$ then find the value of $\cos\theta$ in terms of z.

j) If z = a is a simple pole of f (z) then state the formula for Residue of f (z) at z = a.

2. a) Attempt **any two** of the following. **6**

i) For complex numbers z_1 & z_2 prove that $|z_1\cdot z_2|=|z_1|\cdot|z_2|$ and $\arg(z_1\cdot z_2)=\arg z_1+\arg z_2$.

ii) If $\frac{z-1}{z+i}$ is purely imaginary, find the locus of z.

iii) Solve $x^8-x^4+1=0$.

b) Find all cube roots of unity. **2**

3. Attempt **any two** of the following. 8

a) State Cauchy Riemann equations. Prove that real and imaginary parts of an analytic function satisfy Laplace differential equation.

b) Find an analytic function $f(z)=u+iv$ and express it in terms of z if $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$

c) If $f(z)=\dfrac{3z^4 - 2z^3 + 8z^2 - 2z + 5}{z-i}$ is continuous at z=i, find f (i).

4. a) Attempt **any two** of the following. 6

i) State Cauchy Goursat theorem and verify it for $f(z)=z^2$ round the circle |z| = 1.

ii) Evaluate $\displaystyle\int_{|z|=2} \frac{e^{2z}}{(z-1)^4} dz$

iii) Evaluate $\displaystyle\int_{|z|=4} \frac{dz}{z+2}$

b) Write the expansion of $(1-z)^{-1}$ for |z|<1. 2

5. a) If f (z) is analytic inside and on a closed contour C except at a finite number of singular points then prove that 4

$\displaystyle\int_C f(z)dz = 2\pi i \Sigma R$ where ΣR

denote the sum of the residues at its poles inside C.

b) Evaluate $\displaystyle\int_0^{2\pi} \frac{d\theta}{5+3\sin\theta}$ 4

OR

a) Evaluate $\displaystyle\int_C \frac{3z^2+2}{(z-1)(z^2+9)} dz$ by Cauchy's residue theorem where C is the circle |z-2| = 2. 4

b) Evaluate $\displaystyle\int_0^{2\pi} \frac{d\theta}{5+3\cos\theta}$. 4

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