Seat Number

April 2015

खगोल - 039

* 0 3 1 6 *

MATHEMATICS PAPER - II : MTH - 112

Calculus

(11116)

P. Pages : 3

Time : Two Hours **Max. Marks : 40**

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to the right indicate full marks.
6. use of calculator is not allowed.

1. Attempt **any eight** of the following. **8**

i) Evaluate $\lim_{x \to 2} \frac{x^2 - x - 2}{x^2 - 5x + 6}$

ii) Evaluate $\lim_{x \to 0} \frac{\sin 3x}{x}$

iii) For which value of $C \in \left(-\frac{\Pi}{2}, \frac{\Pi}{2}\right)$ the Rolle's theorem is applicable for the function $F(x) = \cos x$ in $\left[-\frac{\Pi}{2}, \frac{\Pi}{2}\right]$.

iv) State Langrange's Mean Value theorem.

v) Evaluate $\lim_{x \to 1} \frac{\log x}{x - 1}$

vi) Write n^{th} derivative of cos (ax + b).

vii) Write the expansion of e^x

viii) Evaluate $\int_0^{\Pi/2} \sin^8 x \, dx$

ix) Define uniform continuity of function on an interval.

x) State Maclaurin's theorem.

2. a) Attempt **any two** of the following. **6**

i) Prove that every continuous function on closed and bounded interval is bounded.

ii) Examine the continuity of the function.

$$F(x) = \begin{cases} \dfrac{x^2 - 9}{x-3} & \text{for } 0 \le x < 3 \\ 6 & \text{for } x = 3 \\ 8 - \dfrac{18}{x^2} & \text{for } x > 3 \end{cases}$$

at x = 3.

iii) Evaluate $\lim_{x \to 0} (\operatorname{cosec} x)^{\frac{1}{\log x}}$

b) Evaluate $\lim_{x \to 0} \left[\frac{1}{x} - \frac{1}{e^x - 1}\right]$ **2**

3. Attempt **any two** of the following. **8**

i) State and Prove Cauchy's Mean Value theorem.

ii) Verify Langranges Mean Value theorem for the function $F(X) = x(x-1)(x-2)$ in $\left[0, \frac{1}{2}\right]$

iii) Show that

$$\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2} \text{ if } 0 < a < b$$

and hence deduce that

$$\frac{\Pi}{4} + \frac{3}{25} < \tan^{-1}\left(\frac{4}{3}\right) < \frac{\Pi}{4} + \frac{1}{6}$$

4. a) Attempt **any two** of the following. **6**

i) State and Prove Leibnitz's theorem for the n^{th} derivative of product of two functions.

ii) Find the n^{th} derivative of $\frac{x^2+1}{(x-1)(x-2)(x-3)}$

iii) If $y = \sin^{-1} x$ show that $(1-x^2)\cdot y_{n+2} - (2n+1)\, xy_{n+1} - n^2 y_n = 0$

b) Find the n^{th} derivative of $x \log x$. **2**

5. i) Obtain the reduction formula for $\int \frac{\sin nx}{\sin x} dx$, $(n > 1)$ where n is a positive integer. **4**

ii) Obtain by Maclaurin's theorem the first three terms in the expansion of $\log(1 + \sin x)$. **4**

OR

i) State and Prove Taylor's theorem with Langrange's form of remainder after n terms. **4**

ii) Evaluate $\int_0^{\infty} \frac{x^4}{(1+x^2)^5} dx$ **4**

****