April 2015

*0066*

## MATHEMATICS PAPER - II : MTH - 122
## Algebra
### (12116)

P. Pages : 3

Time : Two Hours

Max. Marks : 40

_____

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to the right indicate full marks.

1.     Attempt **any eight** of the following :                                    8

i)     State Division algorithm.

ii)    Define relatively prime integer.

iii)   Define Equivalence Classes.

iv)    Let $A = \{1, 2, 3, 4, 5\}$, the relation R on A defined as aRb iff a < b. Find range of R.

v)     If $\alpha, \beta$ are roots of the equation $ax^2 + bx + c = 0$ then $\alpha + \beta = \text{.......}$ and $\alpha\beta = \text{.......}$ .

vi)    If $\alpha, \beta, \gamma$ are the roots of the cubic equation $2x^3 - 6x^2 + 3x + 1 = 0$ find the value of $\Sigma \alpha\beta\gamma$ .

vii)   Change the signs of the roots of the $3x^8 + 5x^5 - 2x^2 + 4 = 0$

viii) Find the equation whose roots are the reciprocal of the roots of
$$x^3 + 5x^2 - 7x + 8 = 0$$

ix) Define Greatest common divisor.

x) To remove the second term from the equation
$x^3 - 12x^2 + 48x - 72 = 0$, the roots are diminished by .......

2. a) Attempt **any two** of the following :                                   6

i) Use the principle of finite induction show that
$2^n < n!$, for all $n \geq 4$.

ii) If $d = (a, b)$, $a = dx$, $b = dy$, $x, y \in \mathbb{Z}$ then show that $(x, y) = 1$

iii) Find the g. c. d. of 252 and 595 and express it in the form $252m + 595n$.

b) If a/b and b/c then prove that a/c.                                   2

3.    Attempt **any two** of the following :                                   8

i) State and prove Fermat's theorem.

ii) Prepare the composition table for addition and multiplication of residue classes modulo 7.

iii) Define a relation R on $\mathbb{Z}$ by aRb if x - y is an even integer. Prove that R is equivalence relation.

4. a) Attempt **any two** of the following :                                   6

i) If $\alpha, \beta, \gamma$ are the roots of the equation $x^3 - px^2 + qx - r = 0$ find
the value of $\dfrac{1}{\beta^2\gamma^2} + \dfrac{1}{\gamma^2\alpha^2} + \dfrac{1}{\alpha^2\beta^2}$.

ii) Solve the equation $x^3 - 5x^2 - 2x + 24 = 0$, if the product of the roots is 12.

iii) Find the condition that the root of the equation
$x^3 - px^2 + qx - r = 0$ are in A.P.

b) If $\alpha$ and $\beta$ are the roots of $ax^2+bx+c=0$ find the value of $\alpha^3+\beta^3$.    2

5. a) i) Remove the fractional coefficient from the equation    4

$x^3-\dfrac{5}{2}x^2-\dfrac{7}{18}x+\dfrac{1}{108}=0$.

    ii) Find the equation whose roots are the roots of    4

$x^4-x^3-10x^2+4x+24=0$ increased by 2.

<div align="center">OR</div>

a) i) Remove the second term from the equation    4

$x^4+x^3+x-5=0$

    ii) Explain Carden's method of solving the cubic equation.    4

<div align="center">**********</div>