Seat Number

NOV-2015

कात - 037

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MATHEMATICS PAPER - II : MTH - 122

Algebra

(12116)

P. Pages : 3

Time : Two Hours

Max. Marks : 40

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to the right indicate full marks.

1. Attempt **any eight** of the following. 8

i) Define Greatest Common divisor.

ii) State Unique factorization theorem.

iii) Define Euler's function.

iv) Define an equivalence class.

v) If $\alpha, \beta, \gamma, \delta$ are the roots of the equation
$ax^4 + bx^3 + cx^2 + dx + e = 0$
then find the value of $\sum \alpha\beta\gamma\delta$.

vi) If α and β are the roots of the equation $ax^2 + bx + c = 0$ then find
$\alpha^3 + \beta^3$

vii) Change the signs of the roots of the equation
$x^7 + 3x^5 + x^3 - x^2 + 7x + 2 = 0$

viii) Find the equation whose Roots are equal in magnitude but opposite in signs of the foots of
$x^5 + 4x^3 - 6x^2 + 4x - 7 = 0$

ix) State Fermat's theorem.

x) To remove the second term from the equation $x^4 + 20x^3 + 143x^2 + 430x + 462 = 0$, the roots are diminished by .....

2. a) Attempt **any two** of the following. 6

i) If (a, b) = 1 and a|bc then show that a/c.

ii) Use the principle of finite induction, show that $3^{2n+2} - 8n - 9$ is divisible by 64, where $n \in N$.

iii) Find g.c.d. of 75 and 48 and express it in the form 75m + 48n.

b) If a/b and b/a then prove that $a = \pm b$. 2

3. Attempt **any two** of the following. 8

i) Let n be a natural number and a, b, c are integers such that $a \equiv b \pmod{n}$ then show that

a) $(a + c) \equiv (b + c) \pmod{n}$

b) $ac \equiv bc \pmod{n}$

ii) Find the remainder when 8^{401} is divided by 13.

iii) A relation R is defined in the set of integer z by aRb if and only if 7a – 3b is divisible by 4. Show that R is an equivalence relation.

4. a) Attempt **any two** of the following. 6

i) If α, β, γ are the roots of the cubic equation $x^3 + px^2 + qx + r = 0$. Find the value of $\sum \alpha^2$.

ii) Solve the equation $x^3 - 5x^2 - 16x + 80 = 0$, if the sum of two of its roots being equal to zero.

iii) Solve the equation $x^3 - 5x^2 - 2x + 24 = 0$, if the product of two of it's roots is 12.

b) If α and β are the roots of $3x^2 - 4x + 7 = 0$. Find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$ 2

5. a) i) Remove the fractional coefficient from the equation 4

$$x^3 - \frac{5}{2}x^2 - \frac{7}{18}x + \frac{1}{108} = 0$$

ii) Remove the second term from the equation 4

$x^3 - 9x^2 - 4x + 7 = 0$.

OR

a) i) Explain the Descarte's method of solving biquadrate equation. 4

ii) Find the equation whose roots are the roots of 4

$x^4 - 5x^3 + 7x^2 - 17x + 11 = 0$, each diminished by 4.

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