Oct-2014

Seat Number

05502

MATHEMATICS PAPER - II : MTH-242
A) Topics in Differential Equations (New) (24116) / OR
B) Differential Equations and Numerical Methods (New) (24117)

P. Pages : 4

A) Topics in Differential Equations
(New) (24116)

Time : Two Hours **Max. Marks : 40**

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Answer sheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicate full marks.

1. Attempt **any eight** of the following. **8**

a) State the Lipschitz condition.

b) The solution set of $\frac{dx}{x}=\frac{dy}{y}=\frac{dz}{z}$ is

i) $xy=c_1$ and $yz=c_2$ ii) $x=c_1$ and $y=c_2z$
iii) $x=c_1z$ and $y=c_2z$ iv) $y=c_1z$ and $y=c_2x$.

c) Define Pffaffian differential equation in 3 variable.

d) Define Gamma function.

e) The Wronskian of sinx and cosx is.

i) 0 ii) 1
iii) -1 iv) 3.

f) Define simultaneous differential equation of first order.

g) Show that $xdx+ydy+zdz=0$ is exact.

h) Find the value of $\Gamma(\frac{1}{2})$.

i) What is relation between Beta and Gamma function ?

j) Define homogenous equation.

2. a) Attempt **any two** of the following. 6

i) If S defined on the rectangle $|x| \le a, |y| \le b$.. Show that the function $f(x.y) = x\sin y + y\cos x$ satisfy the Lipschitz condition. Find Lipschitz constant.

ii) Two solutions $y_1(x)$ and $y_2(x)$ of the equation $a_0(x)y'' + a_1(x)y' + a_2(x)y = 0, \quad a_0(x) \neq 0, \forall x \in (a,b)$ are linearly dependent then show that their Wronskian is identically zero.

iii) Solve by the method of variation of parameter.
$x^2 y'' + xy' - y = x^2 e^x$.

b) Examine the functions x^2, e^x, e^{-x} for linear independence. 2

3. Attempt **any two** of the following. 8

i) Solve $\frac{dx}{z^2} = \frac{y\,dy}{xz^2} = \frac{dz}{xy}$.

ii) Solve $\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$.

iii) Solve $\frac{dx}{1} = \frac{dy}{3} = \frac{dz}{5z + \tan(y-3x)}$

4 a) Attempt **any two** of the following. 6

i) Show that $(2x + y^2 + 2xz)dx + 2xy\,dy + x^2 dz = 0$ is integrable.

ii) Solve $(x-y)\,dx - x dy + z dz = 0$

iii) Solve $xz^3 dx - z dy + zy dz = 0$.

b) show that $(yz + 2x)dx + (xz - 2z)dy + (xy - 2y)\,dz = 0$ is exact. 2

5. a) 4

i) Evaluate $\int_0^\infty x^7 . e^{-2x^2} dx$

ii) Prove that $B(m,n) = B(m, n+1) + B(m+1, n)$. 4

OR

a) 4

i) Evaluate $\int_0^\infty \sqrt[3]{x^2} \cdot e^{-\sqrt[3]{x}} dx$.

ii) Prove that

$$\Gamma(m)\,\Gamma\left(m + \tfrac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$$ 4

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B) *Differential Equations and Numerical Methods (New) (24117)*

Time : Two Hours **Max. Marks : 40**

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Answer sheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicate full marks.
6. Use of calculator is allowed.

1. Attempt **any eight.** **8**

a) State uniqueness theorem for the initial value problem

$\frac{dy}{dx}=f(x,y) \quad y(x_0)=y_0.$

b) Find Wronskian of $y_1(x)=\sin x$ and $y_2(x)=\sin x-\cos x$

c) Show that e^{3x} and e^{4x} are L.I.

d) Solve $\frac{dx}{yz}=\frac{dy}{zx}=\frac{dz}{zy}$

e) Choose the correct option.

$\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}=\frac{\ell dx+mdy+ndz}{-------}$

i) $\ell+m+n$ ii) 1

iii) 0 iv) $\ell P+mQ+nR$

f) State necessary and sufficient condition for integrability of the pfaffian differential equation $Pdx+Qdy+Rdz=0$

g) Which of the following is true.

i) Every exact equation is integrable.

ii) Every exact equation need not be integrable

iii) Every integrable equation is exact.

iv) The equation is exact iff it is integrable.

h) State Picard's formula for solving the differential equation.

i) Write the Adams – Moulton corrector formula.

j) Choose the correct option

Fourth order Runge Kutta formula is

i) $y_{n+1}=y_0+\frac{1}{2}(K_1+2k_2+2k_3+k_4)$

ii) $y_{n+1}=y_0+\frac{1}{6}(K_1+2k_2+2k_3+k_4)$

iii) $y_{n+1}=y_n+\frac{1}{6}(K_1+2k_2+2k_3+k_4)$

iv) $y_{n+1}=y_0+\frac{1}{2}(k_1+k_2+k_3+k_4)$

2. a) Attempt **any two.** 6

i) If $y_1(x)$ and $y_2(x)$ are any two solutions of $a_0(x)y''(x)+a_1(x)y'(x)+a_2(x)y(x)=0$ then for any two constants k_1 and k_2 show that $k_1y_1(x)+k_2y_2(x)$ is also solution of the given equation.

ii) Check whether the function $f(x,y) = y^{3/4}$ defined on a rectangle $S=\{(x,y)/|x|\le 1, |y|\le 1\}$ satisfy Lipschitz condition or not.

iii) Using method of variation of parameters solve $y''-2y'+y=e^x$.

b) Show that the functions coszx and sinzx are linearly independent. 2

3. Attempt **any two.** 8

i) Solve $\frac{dx}{x+y}=\frac{dy}{x+y}=\frac{dz}{-x-y-2z}$

ii) Solve $\frac{dx}{1}=\frac{dy}{3}=\frac{dz}{5z+\tan(y-3x)}$

iii) Solve $\frac{yz\,dx}{y-z}=\frac{zx\,dy}{z-x}=\frac{xy\,dz}{x-y}$.

4. a) Solve **any two.** 6

i) $yzdx + zxdy - xy(1+2)\,dz = 0$

ii) $yz\log z\,dx - zx\log z\,dy + xydz = 0$

iii) $\left(y^2+z^2-x^2\right)dx - zxydy - zxzdz = 0$

b) Show that the equation. 2

$\left(x^2-yz\right)dx+\left(y^2-zx\right)dy+\left(z^2-xy\right)dz=0$ is exact

5. a) Using modified Euler's method find $y(1\cdot 2)$ for the differential equation $\frac{dy}{dx}=x+y$ under the condition $y = 1$ at $x = 1$. 4

b) Using fourth order Runge Kutta method find. 4

$y(0\cdot 1)$ if $\frac{dy}{dx}=x+y^2$, $y=1$ at $x=0$

OR

a) Using Picards method find value of $y(0\cdot 1)$ given that 4

$\frac{dy}{dx}=y^2+x$, $y(0)=1$

b) If $\frac{dy}{dx}=\frac{1}{x+y}$ $y(0)=2$ $y(0\cdot 2)=2.0933$ $y(0\cdot 4)=2.1755$ 4

$y(0\cdot 6)=2.2493$ find $y(0\cdot 8)$

Using Milne's method.

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