April - 2016



Seat Number

* 0 8 7 9 *

## MATHEMATICS PAPER - II : MTH-122
## Theory of Numbers and Equations
## (111202)

**P. Pages : 4**

**Time : Two Hours** **Max. Marks : 60**

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicates full marks.

1. a) Attempt **any six** of the following. 6

i) If $a \equiv b(\text{mod } n)$ then
- a) $a + b = kn$ for some integer k
- b) $a - b = kn$ for some integer k
- c) $a - b = n$
- d) $a + b = n$

ii) If $\alpha, \beta, \gamma$ are the roots of $x^3 + px^2 + qx + r = 0$ then $\alpha\beta + \beta\gamma + \gamma\alpha$ equal ---
- a) $-p/q$ b) $-p$
- c) $q$ d) $-q$

iii) Given integer a and b with b > 0, then
- a) $a = bq + r \quad 0 \le r < b$ b) $a = qb + r \quad 0 \le r < b$
- c) $a = bq + r \quad 0 \le r < 1$ d) None of them

iv) The number $\sqrt{2}$ is -----
- a) Rational b) Irrational
- c) n real roots d) None of them

v) If a and b are relatively prime then
- a) a/b b) b/a
- c) g.c.d (a,b) = 1 d) None of them

vi) If $n > 2$, $\phi(n)$ is ---

a) Prime number
b) Even number
c) Odd number
d) None of them.

vii) If $\alpha, \beta$ are the roots of eq$^n$ $ax^2 + bx + c = 0$ then $\alpha + \beta$

a) $b/a$
b) $-b/a$
c) $a/c$
d) $c/a$

viii) Let g.c.d (a, b) = d then

a) $d/a$ and $d/b$
b) $d/a$ only
c) $d/b$ only
d) None of them

b) Attempt **any six** of the following. 6

i) Define congruence relation modulo n.

ii) Define Equivalence relation.

iii) State Fermat's theorem.

iv) State Euler's theorem.

v) Define Greatest common divisor.

vi) Change the sign of the roots of eq$^n$ $x^7 + 5x^5 - x^3 + x^2 + 7x + 3 = 0$.

vii) If $Z_6 = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}\}$ is the set of all residue classes modulo 6. Find $\bar{4} +_6 \bar{5}$.

viii) Define Division Algorithm.

2. Attempt **any six** of the following. 12

i) If p is prime and $a^2 \equiv b^2 \pmod p$ show that $p/a+b$ or $p/a-b$

ii) Write down the elements of $Z_{11}^1$.

iii) To remove the second term from the equation
$x^4 - 4x^3 - 18x^2 - 3x + 2 = 0$ the roots are diminished by -----

iv) If $\alpha, \beta, \gamma$ are the roots of equation $x^3 - 5x^2 - 2x + 24 = 0$ then
$\Sigma\alpha\beta = .....$ and $\Sigma\alpha\beta\gamma = .....$

v) Find g.c.d of 616 and 427.

vi) Find the equation whose roots are the reciprocal of the roots of
$x^3 + 5x^2 - 7x + 18 = 0$.

vii) If $a/b$ and $a/c$ then prove that $a/{bx+cy}$ for all $x, y \in Z$.

viii) Find the equation whose roots are negative of the roots
$5x^4 + 4x^2 - 7x + 5 = 0$.

ix) Let $A = \{3, 4, 5\}$ and $B = \{2, 4, 6\}$ and R be relation from A to B defined by $xR_y$ if $x < y$. Find R.

3. Attempt **any four** of the following. 12

i) Using Euler's theorem, find the remainder when $15^{27}$ is divided by 8.

ii) Show that $\sqrt{7}$ is not a rational number.

iii) Show by Induction $7^n + 2$ is divisible by 3.

iv) Let '~' be equivalence relation on a non empty set A and $a, b \in A$ then
$b \in [a]$ iff $[a] = [b]$

v) Prepare the composition table for addition and multiplication of residue classes modulo 6.

vi) Define a relation R on Z by $aR_b$ if $a - b$ is an even integer. Prove that R is an equivalence relation.

4. Attempt **any three** of the following. 12

i) Solve the equation $x^3 - 5x^2 - 2x + 24 = 0$ if the product of two of the roots is 12.

ii) If $\alpha, \beta, \gamma$ are the roots of the equation $x^3 - px^2 + qx - r = 0$, find the value of $\frac{1}{\beta^2\gamma^2} + \frac{1}{\gamma^2\alpha^2} + \frac{1}{\alpha^2\beta^2}$.

iii) Write down the relation between roots and coefficient of Biquadratic equation $ax^4 + bx^3 + cx^2 + dx + e = 0$.

iv) Solve the equation $x^3 - 6x^2 + 3x + 10 = 0$ if the roots are in arithmetical progression.

v) If $\alpha$ and $\beta$ are roots of $3x^2 - 4x + 7 = 0$. Find the value of

i) $\alpha^2 + \beta^2$

ii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

5. Attempt **any two.** **12**

i) Find the equation whose roots are those of $3x^3 - 2x^2 + x - 9 = 0$ each diminished by 5.

ii) Remove the fractional coefficients from the equation

$$x^4 + \frac{3}{10}x^2 + \frac{13}{25}x + \frac{77}{1000} = 0$$

iii) Remove the second term from the equation
$x^4 + x^3 + x - 5 = 0$

**OR**

Explain the Descartes method of solving biquadratic equation.

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