Nov-2016

Seat Number



* 1 0 3 6 *

# Mathematics Paper - I : MTH - 111
## Matrices
## (111101)

P. Pages : 4

Time : Two Hours

Max. Marks : 60

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicates full marks.

**1.** a) Attempt any **six** of the following. **6**

i) Inverse of a matrix if it exists, is ...........
   a) square  b) unique
   c) not unique  d) none of these

ii) If A, B are non-singular matrices of the same order then adj(AB)=......
   a) adjA.adjB  b) adjB.adjA
   c) adj(BA)  d) none of these

iii) The rank of zero matrix is ..........
   a) 0  b) 1
   c) -1  d) none of these

iv) $[E_{13(-2)}]^{-1}$ : The elementary matrix obtained by using elementary transformation.........
   a) $R_{13(2)}$  b) $R_{13(-2)}$
   c) $R_{13}$  d) none of these

v) A system AX = B is consistent then $\rho(A)$.......$\rho[A, B]$
   a) is equal to  b) is not equal to
   c) greater than to  d) none of these

vi) If $Ax = \lambda x$ then $\lambda$ is called ..........
   a) Eigen value  b) Eigen vector
   c) Characteristic vector  d) none of these

vii) Inverse or orthogonal matrix is..............
   a) Proper orthogonal   b) Improper orthogonal
   c) Orthogonal   d) none of these

viii) If r is rank & s is the index, n is number of variables of quadratic form is negative semidefinite if ............
   a) $r < n, s = r$   b) $r < n, s = o$
   c) $r > n, s = r$   d) none of these

b) Attempt **any six** of the following. **6**

i) Define minor of an element of matrix.

ii) If $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$ find adj A

iii) Elementary transformation ........... change the rank of matrix.

iv) Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ find $E'_{21(5)}$.

v) State Cayley Hamilton theorem.

vi) Define characteristic value of corresponding matrix A.

vii) Define Improper orthogonal matrix.

viii) Write quadratic form in three variables.

**2.** Attempt **any six** of the following. **12**

i) If A is non-singular matrix of order n & k is a non-zero scalar then prove that $(kA)^{-1} = \frac{1}{k}A^{-1}$

ii) Let $A = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 4 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ find the contactors $A_{31}$ & $A_{33}$.

iii) Define rank of the matrix.

iv) Let A, B be two matrices of same order with $\rho(A) = \rho(B)$ then prove that $A \sim B$.

v) Define linearly independent & linearly dependent solution of system $AX = 0$.

vi) Find eigen value of matrix $A = \begin{bmatrix} 9 & -7 \\ 3 & -1 \end{bmatrix}$

vii) Show that an orthogonal matrix A is proper orthogonal if
$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

viii) Find the matrix of quadratic form $5x^2 + 10xy + 3y^2$

ix) Define singular & non-singular matrix.

**3.** Attempt **any four** of the following. **12**

i) If A is a non-singular matrix of order n then prove that
$\operatorname{adj}(\operatorname{adj} A) = |A|^{n-2} A$

ii) If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ verify that $A \cdot (\operatorname{adj} A) = (\operatorname{adj} A) \cdot A = |A|\, I$.

iii) Prove that every non-singular matrix can be expressed as a product of finite number of elementary matrices..

iv) Reduce the matrix $A = \begin{bmatrix} 1 & 5 \\ -2 & 1 \\ 3 & 4 \end{bmatrix}$ to its normal form & find $\rho(A)$.

v) Examine for consistency the following system of equations
$-2x + 5z = 2$
$5x + y + 2z = 3$
$2x + y + 7z = -2$

vi) Prove that product of two orthogonal matrices of same order is orthogonal.

**4.** Attempt **any three** of the following. **12**

i) If A is non-singular matrix & n is natural number then prove that
$(A^n)^{-1} = (A^{-1})^n$, $n \in N$.

ii) If find inverse of matrix by using adjoint method

where $A = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 2 & 0 \\ 2 & 1 & 1 \end{bmatrix}$

iii) Determine the value of x that will make the matrix A given of rank 3.

where $A = \begin{bmatrix} x & x & 1 \\ 1 & x & x \\ x & 1 & x \end{bmatrix}$

iv) Investigate for what values of $\lambda$ & $\mu$ the following system of equations

$x + 3y + 2z = 2$

$2x + 7y - 3z = -11$

$x + y + \lambda z = \mu$

have an infinite number of solutions.

v) Write down the quadratic form corresponding to the symmetric matrix A where

$A = \begin{bmatrix} 0 & 5 & -1 \\ 5 & 1 & 6 \\ -1 & 6 & 2 \end{bmatrix}$

5. Attempt **any two** of the following. **12**

i) Prove that A is matrix of rank r then there exists non-singular matrices P and Q such that $PAQ = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$

Find non-singular matrices P & Q such that PAQ is in normal form of A where $A = \begin{bmatrix} 2 & 6 \\ 1 & 3 \\ 3 & 9 \end{bmatrix}$

ii) Define elementary matrix & express a non-singular matrix

$A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ as product of elementary matrices.

iii) Define :

a) Canonical form
b) Index
c) Signature
d) Positive definite
e) Negative definite
f) Positive semi definite

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