Seat Number

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MATHEMATICS PAPER - III : MTH-113
A) Geometry (111103) /
B) Discrete Mathematics (111104)

P. Pages : 8

A) Geometry (111103)

Time : Two Hours **Max. Marks : 60**

Instructions to Candidates :
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2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to the right indicates full marks.

1. a) Attempt **any six.** **6**

i) If by rotating the axes through an angle 45° then the equations of rotation are..........

a) $x=\frac{x'+y'}{\sqrt{2}}, y=\frac{x'-y'}{\sqrt{2}}$ b) $x=\frac{x'-y'}{\sqrt{2}}, y=\frac{x'+y'}{\sqrt{2}}$

c) $x=\frac{x'+y'}{2}, y=\frac{x'-y'}{2}$ d) $x=\frac{x'-y'}{2}, y=\frac{x'+y'}{2}$

ii) The equation $16x^2-24xy+9y^2-6x-8y-1=0$ represents.......

a) Ellipse b) Hyperbola

c) Parabola d) Circle

iii) The number of tangent lines at a point on the sphere are..........

a) Two b) Three

c) Infinite d) None of these

iv) Equation of a sphere having centre origin and radius a is.......

a) $x^2+y^2+2x=a^2$ b) $x^2+y^2+z^2-ax=0$

c) $x^2+y^2+z^2=a^2$ d) None of these

v) If the cone passes through the three axis then the vertex of the cone is......

a) 0 (0,0,0) b) V (α, β, γ)

c) A (0,0, a) d) B (0, a, 0)

vi) The constant angle between the axis and the generator of a right circular cone is called as........

a) Vertical b) Semi-vertical

c) Major d) None of these

vii) The direction ratios of the generator of the right circular cylinder whose axis is $\frac{x}{2}=\frac{y}{3}=\frac{z}{5}$ are..........

a) 2, 3, 5 b) 1, 2, 3

c) 3, 4, 5 d) None of these

viii) In the enveloping cylinder generators touch a given surface and are..... to a given straight line.

a) Intersect b) Parallel

c) Perpendicular d) None of these

b) Attempt **any six.** 6

i) State whether true or false.
The equation of the conic is a linear equation in x and y.

ii) Fill in the blanks.
The equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a circle if--

iii) Define Great circle.

iv) State the condition that two spheres cut orthogonally.

v) Fill in the blanks.
If a, b, c are d.r.s. of any generator of the cone f(x,y,z)=0 with vertex origin then f (a, b, c)= ......

vi) Define Enveloping cone of a sphere.

vii) Define a Right circular cylinder.

viii) State whether true or false.
The guiding circle of an enveloping cylinder of sphere is a great circle of the sphere.

2. Attempt **any six.** 12

i) If the origin is shifted to the point (3,-4) the axes remaining parallel to the original set of axes. Find the co-ordinates of the point (-2,3) with respect the new set of axes.

ii) Find θ through which the axes should be rotated in order to remove the xy term from the equation.
$7x^2+12xy-5y^2+4x+3y-2=0$.

iii) Identify the conic and find the centre of the conic represented by
$x^2-xy+y^2+4x-5y-2=0$.

iv) Find the centre and radius of the sphere.
$x^2+y^2+z^2-12x+4y-6z+48=0$.

v) Find the equation of the sphere passing through 0 (0, 0, 0), A (a, 0, 0), B (0, b, 0) and c (0, 0, c)

vi) Obtain the equation of a cone which passes through the three axis.

vii) If the line $\frac{x}{2}=\frac{y}{-1}=\frac{z}{3}$ is the generator of the cone
$x^2+y^2+z^2+axy-xz=0$. Find 'a'.

viii) Find the equation of the axes of the right circular cylinder whose guiding circle is the circle passing through the points A (a, 0, 0), B (0, a, 0) and C (0, 0, a).

ix) Find the equation of the cylinder whose generators are parallel to the z-axis and intersect the curve $ax^2+by^2+cz^2=1$, $lx+my+nz=p$.

3. Attempt **any four.** 12

i) If the origin is shifted to the point (n, 2), find the value of n so that the new equation of the locus given by the equation $x^2+4x+3y-5=0$ will not contain a first degree terms in x.

ii) If (x, y) and (x', y') be the co-ordinates of the same point referred to two sets of rectangular axes with the same origin. If px+qy where p and q are independent of x, y becomes $p'x'+q'y'$. Show that
$p^2+q^2=p'^2+p'^2$.

iii) Find the equation of the tangent plane to the sphere.
$x^2+y^2+z^2-2x-y-z-5=0$ at the point (1, 1, -2).

iv) Prove that the spheres $x^2+y^2+z^2+7x+10y-5z+12=0$ and $x^2+y^2+z^2-4x+6y+4=0$ intersect orthogonally.

v) Find the equation of the right circular cone with vertex at the origin and which passes through the point (1, 1, 2) and which has its axis as the line 6x = -3y = 4z.

vi) Obtain the equation of cylinder whose generators are parallel to $\frac{x}{1}=\frac{y}{1}=\frac{z}{1}$ and whose guiding curve is $x^2+2y^2=1, z=3$.

4. Attempt **any three.** **12**

i) Show that equation of a conic is a second degree equation in x and y.

ii) Transform the equation $x^2+4xy+y^2-2x+2y-6=0$ when the origin is shifted to the point (-1, 1) and then the axes are turned through an angle of 45°.

iii) Find the equation of the sphere for which the circle $x^2+y^2+z^2-3x+4y-2z-5=0, 5x-2y+4z+7=0$ is a great circle.

iv) Prove that the equation of the cone, whose vertex is the origin and base curve $z=k, f(x,y)=0$ is $f\left(\frac{xk}{2}, \frac{yk}{2}\right)=0$.

v) Find the equation of enveloping cylinder of the sphere $x^2+y^2+z^2-2x+3z+1=0$ whose generators are parallel to $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$.

5. Attempt **any two.** **12**

i) Find the condition that the plane lx+my+nz=p touches the sphere $x^2+y^2+z^2=a^2$ and find the point of contact.

ii) Show that every homogeneous equation in x, y, z represents a cone with vertex at the origin.

iii) Find the equation of cylinder whose generators intersect the guiding curve $ax^2+2hxy+by^2+2gx+2fy+c=0, z=0$.

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B) Discrete Mathematics (111104)

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Max. Marks : 60

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1. a) Attempt **any six** of the following. **6**

i) -----is regular graph of degree 6.

a) K_7 b) $K_{3,2}$

c) K_5 d) $K_{5,1}$

ii) If G is a regular graph of degree r on n vertices then $\overline{G}$ is regular graph of degree-----

a) n+r-1 b) n+r+1

c) n-r+1 d) n-r-1

iii) If G be a graph with P vertices, q edges and K components then ------

a) $q \geq n-k$ b) $q \geq n+1$

c) $q \leq n-1$ d) $q \leq n+1$

iv) A graph $K_{m,n}$ is Hamiltonian if and only if------

a) m<n b) m>n

c) m=n d) none of these.

v) Chromatic number of $K_{2,3}$ is -----

a) 2 b) 3

c) 5 d) 1

vi) For any graph G with P vertices and q edges nullity of G=-------

a) q+p+1 b) q-p+1

c) q-p-1 d) q+p-1

vi) Let G be a planar graph with 16 edges, 9 faces then number of vertices in G is ---------

a) 9 b) 7

c) 25 d) 16

viii) A complete graph K_n is tree if n=?

a) 2 b) 3

c) 4 d) 5

6

b) Attempt **any six** of the following.

i) Degree of Isolated vertex is------

a) 0 b) 1 c) 3 d) 4

ii) Define odd vertex

iii) Define a path in graph.

iv) Define a bridge in graph.

v) Define geometrical dual of graph G

vi) Draw Kuratowski's first graph.

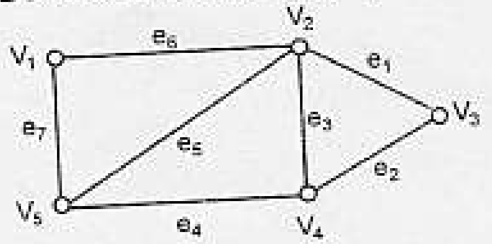
vii) Define radius and diameter of tree.

viii) Define terminal node in tree.

12

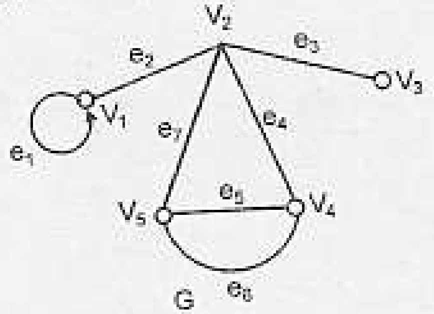
2. Attempt **any six** of the following.

i) Describe the following graph in the form of vertices and edges.



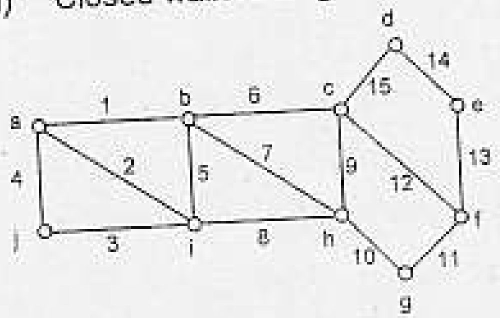
ii) State Handshaking lemma.

iii) Find two spanning subgraphs of graph G given below.

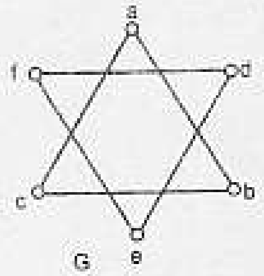


iv) In graph G given below find

i) Closed walk of length 6. ii) Open trail of length 6.



v) Is following graph G connected ? Justify.



G

vi) Draw directed graph G(V,E)

Where V = {u, v, w, z} and $E = \left\{ \overrightarrow{uz}, \overrightarrow{vw}, \overrightarrow{vu}, \overrightarrow{ww}, \overrightarrow{zu} \right\}$

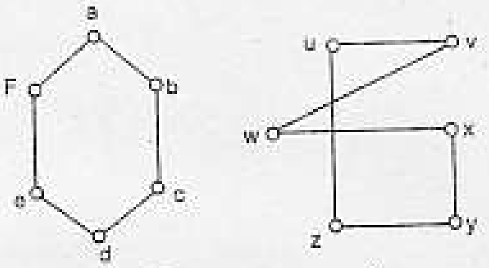
vii) Define geometrical dual of plane graph G.

viii) Define rooted and binary trees.

ix) Draw two non isomorphic trees on 6 vertices.

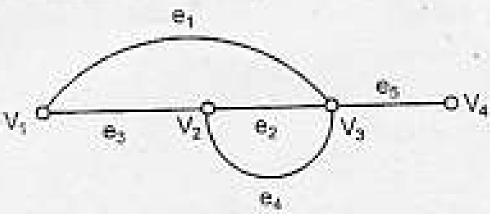
3. Attempt **any four** of the following. **12**

i) Are the following graphs isomorphic? Justify.

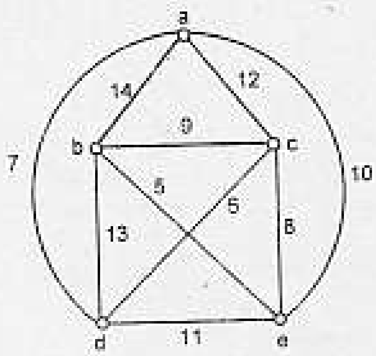


G_1 G_2

ii) Find four distinct spanning subgraphs of the following graph.



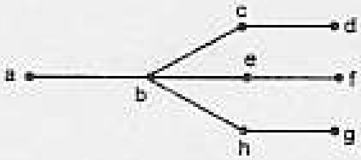
iii) Using nearest neighbor method find Hamiltonian cycle and its total distance for the graph given below



iv) If G is a simple connected nontrivial graph then prove that $K(G) \leq \lambda(G)$.

v) Give an example of a connected graph has
  a) Neither Eulerian circuit nor Hamiltonian circuit.
  b) Eulerian circuit but no Hamiltonian circuit.
  c) Hamiltonian circuit but no Eulerian circuit.

vi) Find the leaves and branch nodes of the following trees.



4. Attempt **any three** of the following **12**

i) Does there exists a simple graph with 7 vertices, one of which has degree 2, two have degree 3, three have degree 4 and the remaining vertex has degree 5? Justify.

ii) Define a) Addition of vertex
  b) Addition of an edge
  c) Fusion of vertices.

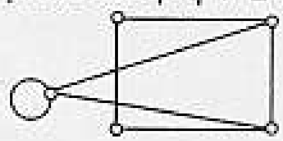
iii) Show that K_5 is not planar graph.

iv) Write short note on "Konigsberg's seven bridges problem".

v) A tree has 2n pendant vertices, 3n vertices of degree 2, and n vertices of degree 3. Determine number of vertices and edges in a tree.

5. Attempt **any two** of the following. **12**

i) If G is connected plane graph with P vertices, q edges and r faces then prove that p-q+r=2 verify for graph G.



G

ii) Find graph G such that graph G with
  a) $K(G) = 3, \lambda(G) = 4, \delta(G) = 5$
  b) Where $\lambda(G) = 0$
  c) Where $\lambda(G) = 1$

iii) Prove that a tree with two or more vertices has at least two leaves.

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