April 2015

Seat Number

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खगोल - 036 / 037

MATHEMATICS PAPER – III : MTH - 113
(A) Co-ordinate Geometry (11117) OR / (B) Graph Theory (11118)

P. Pages : 7

(A) Co-ordinate Geometry (11117)

Time : Two Hours **Max. Marks : 40**

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicate full marks.
6. Use of calculator is not allowed.

1. Attempt **any eight.** **8**

i) Where is the origin shifted when the new coordinates of (5, -1) are (3, 2) ?

ii) State the formula for θ, through which the axes should be rotated so as to remove the term in xy from the expression $ax^2+2hxy + by^2 + 2gx + 2fy+c$.

iii) State the conditions that the equation $ax^2+2hxy + by^2 + 2gx + 2fy+c=0$ represents ellipse.

iv) Find the radius of the sphere $x^2+y^2+z^2-3x-4y+5z+1=0$.

v) Define 'Right circular cone'.

vi) State the equation of the cone passing through three axes.

vii) Define a normal section of the right circular cylinder.

viii) State the conditions that two spheres are externally touching.

ix) Find the equation of the sphere whose diameter has the endpoints (1, 0, 2) and (-1, 3, 0).

x) Define guiding curve of the cylinder. 6

2. a) Attempt **any two.**

i) Prove that every general equation $Ax^2+Ay^2+2Gx+2Fy+C=0$ represents a circle. Find its centre and radius.

ii) If the origin is shifted at (3, k), the transformed equation of locus given by $2y^2+4x-6y+7=0$ does not contain the first degree term in y then find the value of k.

iii) Find the new equation of the locus given by $x^2+4yx+y^2=0$ when the axes are rotated through 45°. Identify the nature of the locus.

b) Find the new equation of the locus given by $x^2+4x-2y+6=0$ when the origin is shifted at (-2, -1). 2

8

3. Attempt **any two.**

i) Find the condition that the plane $lx+my+nz = p$ touches the sphere $x^2+y^2+z^2=a^2$. Also find the point of contact.

ii) Find the equation of the sphere centred at (3,2,1) and touching the plane $2x-2y+z+7=0$.

iii) Show that the spheres $x^2+y^2+z^2+6y+2z+8=0$ and $x^2+y^2+z^2+6x+8y+4z+20=0$ are orthogonal.

6

4. a) Attempt **any two.**

i) Show that the equation of the cone with vertex at origin is homogenous.

ii) Find the equation of the right circular cone with vertex at (2,-1,4) ; semi vertical angle $\text{Cos}^{-1}(4/\sqrt{6})$ and having axis with direction ratios 1, 2, -1.

iii) Find the equation of the cone with vertex at the origin and having the guiding curve.
$x^2+y^2+z^2+4x+3y+7=0$; $3x-y+4z = 2$.

b) State the condition that the general equation $f(x,y,z) \equiv ax^2+by^2+cz^2+2fyz+2gzx+2hxy+2ux+2vy+2wz+d=0$ represents a cone. and also state the equations whose solution is the vertex. **2**

5. a) i) Find the equation of the cylinder whose generators have direction cosine ℓ, m, n and which passes through $x^2+z^2 = 1$ in zox plane. **4**

ii) Find the equation of cylinder whose generators pass through $x+y+z=1$; $x^2+y^2+z^2=4$ and parallel to $\frac{x}{2}=\frac{y}{-1}=\frac{z}{2}$. **4**

OR

a) i) Find the equation of the right circular cylinder whose axis is $\frac{x}{2}=\frac{y}{3}=\frac{z}{6}$ with radius 4. **4**

ii) Find the equation of cylinder whose generators intersect the plane curve $2x^2+3y^2=1$, $z = 0$ and parallel to $x=2y=3z$. **4**

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(B) Graph Theory
(11118)

Time : Two Hours **Max. Marks : 40**

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1. Attempt **any eight** of the following. **8**

i) Define 'degree of a vertex'.

ii) How many total number of edges in K_n are ?

iii) Give an example of a connected graph which is neither an eulerian nor a Hamiltonian.

iv) Define weighted graph.

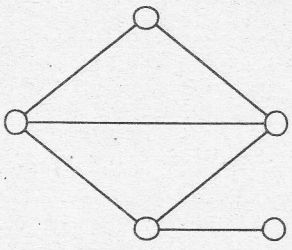
v) A complete graph $K_{m,n}$ is Hamiltonian iff.

a) $m > n$ b) $m \neq n$

c) $m = n$ d) $m < n$

vi) State Euler's formula for planar graph.

vii) Determine chromatic number of the following graph.



viii) Find a tree on 6 vertices having exactly three leaves.

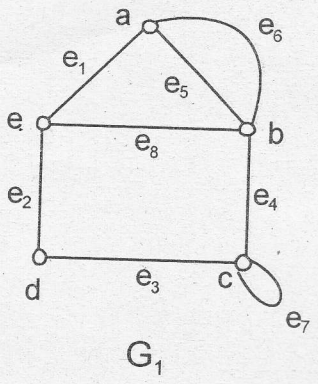
ix) Define 'fundamental cut set'.

x) Define 'spanning tree'.

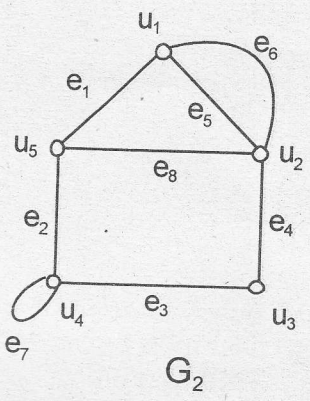
2. a) Attempt **any two** of the following. 6

i) If G is self complementary graph on n vertices then show that n is of the type 4k or 4k+1 for some integer K.

ii) Show that two graphs G_1 and G_2 given below are not isomorphic.



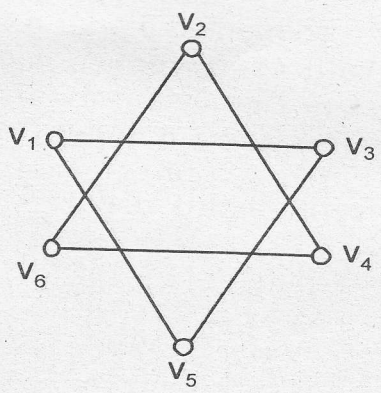
G_1



G_2

iii) Does there exist a regular graph of degree 5 on 7 vertices ? Justify.

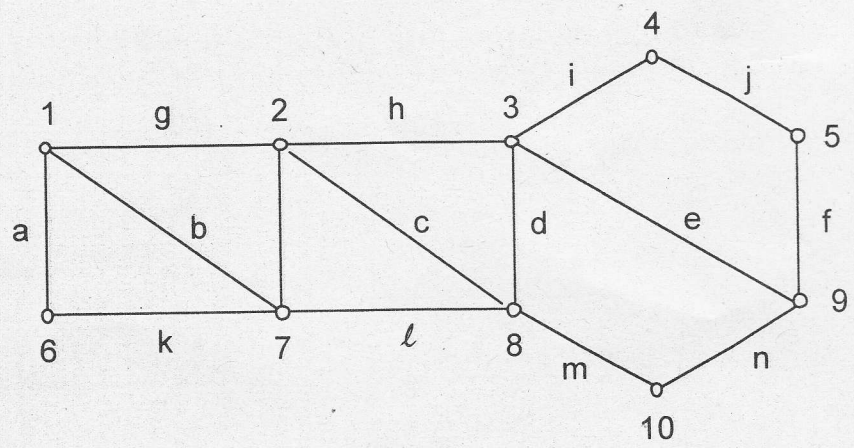
b) Find complement of the following graph. 2



3. Attempt **any two** of the following. 8

i) Let G = (V, E) be a simple graph with k-component and |v| = n, |E| = m then prove that $m \geq n-k$.

ii) For the following graph G



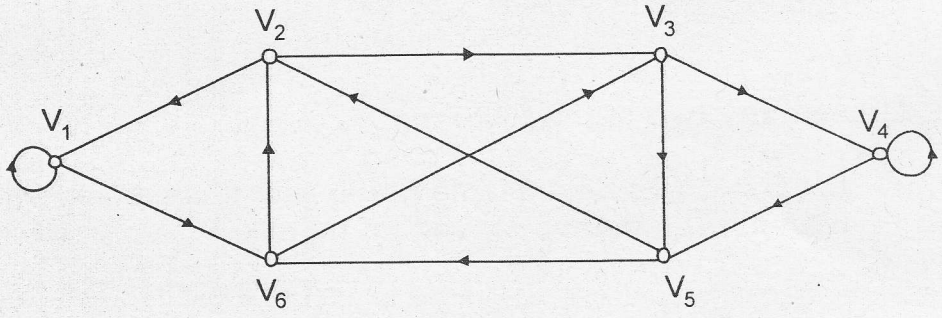
Find :
a) A closed walk of length 8.
b) Distance between 6 and 9.
c) Three cut sets

iii) Construct the graph in which K (G) < λ (G) < δ (G).

4. a) Attempt **any two** of the following. **6**

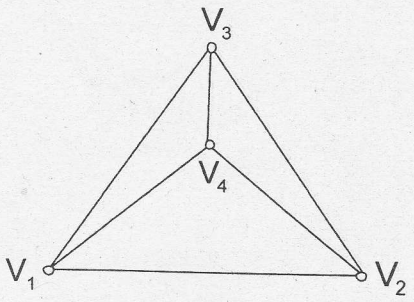
i) Let G be a 2 – connected planer graph without a triangle and if G has p – vertices, q – edges then prove that $q \leq 2p-4$.

ii) Find the indegree and out degree of each vertex in following digraph and verify that $\sum_{i=1}^{n} d^{+}(v_i) = \sum_{i=1}^{n} d^{-}(v_i)$



iii) Find number of edges in a simple planar graph with 16 vertices and 20 faces.

b) What is the dual of the following graph ? 2



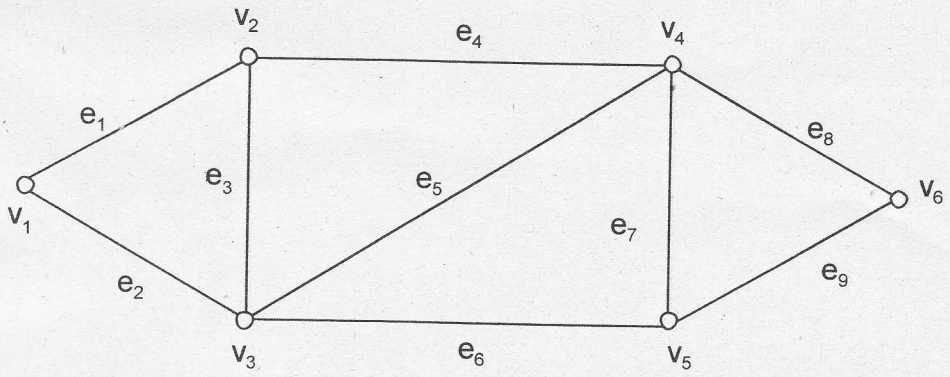
5. a) i) Prove that a tree with n vertices must have (n-1) edges. 4

ii) Draw all possible non – isomorphic trees on 9 – vertices. 4

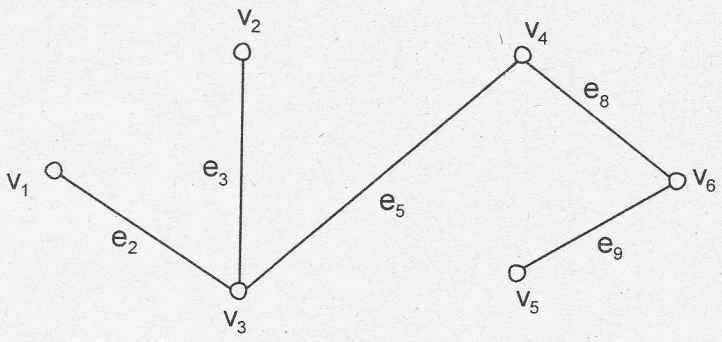
OR

a) i) Prove that every tree is a bipartite graph. Which trees are complete bipartite graphs ? 4

ii) Find the fundamental cutset of a graph G with respect to given spanning tree T. 4



G.



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