Seat Number

Nov-2015

0111

कणगा - 006

MATHEMATICS PAPER - I : MTH - 111
Matrices
(111101)

P. Pages : 4

Time : Two Hours

Max. Marks : 60

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicates full marks.

1. a) Attempt **any six** of the following. 6

i) If A is square matrix of order n and k is non-zero scalar then $|kA| = ...$

a) $k^n |A|$ b) $k^{n-1} |A|$
c) $k |A|$ d) None of these

ii) If A is a non-singular matrix of order n, then adj A is.....matrix.

a) singular b) non-singular
c) not square d) None of these

iii) If A~B then order of B is .....

a) equal to order of A b) greater than order of A
c) less than order of A d) None of these

iv) Eij(k) : The elementary matrix obtained by using elementary transformation .....

a) Rij(k) b) Rij(-k)
c) Rij d) None of these

v) A system AX = B is consistent iff $\rho(A) = ...$

a) $\rho(A)$ b) $\rho(B)$
c) $\rho[A, B)$ d) None of these

vi) $|A-\lambda I| = 0$ is called ..... of A
   a) characteristic equation
   b) Eigen equation
   c) non-characteristic equation
   d) None of these

vii) An orthogonal matrix A is called proper orthogonal of $|A|$ = ...
   a) 0 b) 1
   c) -1 d) None of these

viii) If r is rank, S is index, n is number of variables of quadratic form is positive definite if .....
   a) r = n = s b) r > n > s
   c) r = s & n > s d) None of these

b) Attempt **any six** of the following. 6

i) Define Inverse of matrix.

ii) If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 0 & 1 \\ 1 & -1 & 2 \end{bmatrix}$ find cofactor A_{23}

iii) Define equivalent matrices.

iv) Let $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ find $E'_{23(-5)}$

v) Define non-trivial solution of system of equation $AX = 0$.

vi) Define eigen value of corresponding matrix A.

vii) Define congruent matrix.

viii) Define rank of quadratic form.

2. Attempt **any six** of the following. 12

i) Define Adjoint of matrix & state formula for cofactor Aij.

ii) If A is non-singular matrix & B, C are matrices such that AB = BC show that B = C.

iii) Define normal form of matrix.

iv) Find rank of matrix A if $A=\begin{bmatrix}2 & 3 & 1\\ -1 & 2 & 1\\ -2 & 4 & 1\end{bmatrix}$

v) Define Linearly dependent & linearly independent solution of system of AX = 0.

vi) Find Eigen value of matrix $A=\begin{bmatrix}3 & -5\\ 7 & 8\end{bmatrix}$

vii) Verify the matrix A is orthogonal where $A=\begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{bmatrix}$

viii) Find the matrix of the quadratic form $2x^2+6xy+y^2$.

ix) Define skew symmetric matrix & symmetric matrix.

3. Attempt **any four** of the following. **12**

i) Prove that Inverse of a matrix if it exists, is unique.

ii) Let $A=\begin{bmatrix}3 & -3 & 4\\ 2 & -3 & 4\\ 0 & -1 & 1\end{bmatrix}$
show that A(adj A) is an identity matrix.

iii) If A is matrix of rank r then there exists a non-singular matrix P & Q such that $PAQ=\begin{bmatrix}I_r & 0\\ 0 & 0\end{bmatrix}$

iv) Reduce the matrix $A=\begin{bmatrix}1 & 2 & 3\\ 3 & 1 & 2\end{bmatrix}$ to its normal form & find $\rho(A)$.

v) Examine for consistency the following system of equations.
$x+z=2$
$-2x+y+3z=3$
$-3x+2y+7z=4$

vi) Prove that the determinant of orthogonal matrix is ± 1.

4. Attempt **any three** of the following. 12

i) For any square matrix A, prove that
$A(\text{adj } A) = (\text{adj } A)A = |A| I$

ii) Find Inverse of the matrix A, if it exist where $A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$

iii) Compute the following matrix for the elementary matrices of order 3
$[E_{3(-2)}]^{-1} [E_{13(2)}] [E'_{23(-1)}]$

iv) Show that the system of equation
$ax + by + cz = 0$
$bx + cy + az = 0$
$cx + ay + bz = 0$
has non-trivial solution iff a+b+c = 0 or a=b=c.

v) Find the matrix of quadratic form
$x_1^2 - 2x_2^2 - 3x_3^2 + 4x_1x_2 + 6x_1x_3 - 8x_2x_3$ Also find the rank.

5. Attempt **any two.** 12

i) Prove that the rank of product of two matrices can not be exceed the rank of either matrix and verify that $\rho(AB) \leq \min \{\rho(A), \rho(B)\}$
where $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ & $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

ii) State the Cayley Hamilton theorem and verify that for the matrix
$A = \begin{bmatrix} 1 & 2 & 0 \\ -3 & -2 & 1 \\ 1 & 3 & -1 \end{bmatrix}$

iii) Reduce the quadratic form
$x_1^2 + 2x_2^2 + 2x_1x_2 - x_1x_3 + 2x_2x_3$
to its canonical form, find rank, index & signature.

****