April 2015

खगोल - 007

Seat Number

* 0 0 2 9 *

MATHEMATICS PAPER - I : MTH - 111
Theory of Matrices
(11115)

P. Pages : 3

Time : Two Hours **Max. Marks : 40**

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to the right indicate full marks.
6. Use of calculator is not allowed.

1. Attempt **any eight** of the following : **8**

i) If $A=\begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix}$, Find adj A

ii) If A, B are two square matrices of same order such that $A\cdot B=I$ then B = ?

iii) If $A=\begin{bmatrix}2 & 6\\ 3 & x\end{bmatrix}$ and $\rho(A)=1$, then find the value of x.

iv) Write down inverse of an elementary matrix $E_2(7)$ of order three.

v) Define homogeneous system of linear equations.

vi) State Cayley - Hamilton theorem.

vii) Write down the quadratic form of the matrix

$$A=\begin{bmatrix}1 & 2\\ 2 & 2\end{bmatrix}$$

viii) Show that the matrix $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ is proper orthogonal matrix.

ix) State the rank of matrix A if A is non - singular matrix of order 8.

x) Define signature of a quadratic form.

2. a) Attempt **any two** of the following : **6**

i) If A and B are non - singular matrices of the same order then prove that $adj(A.B) = adjB.\ adjA$

ii) Find the inverse of the matrix A by using adjoint method

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

iii) Verify that $A.adjA = adjA.A = |A|.I$ where $A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$

b) Write down the inverse of an elementary matrix $E_{13}(5)$ of order 3. **2**

3. Attempt **any two** of the following : **8**

i) Prove that every non - singular matrix can be expressed as a product of a finite number of elementary matrices.

ii) Reduce the matrix A to its normal form and find its rank where

$$A = \begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & 2 & 3 & 4 \\ 3 & 4 & 5 & 2 \end{bmatrix}$$

iii) Obtain non - singular matrices P and Q such that PAQ is in the normal form, where $A = \begin{bmatrix} 2 & 6 \\ 1 & 3 \\ 3 & 9 \end{bmatrix}$

Also find the rank of matrix A.

4. a) Attempt **any two** of the following : **6**

i) Find eigen values and eigen vectors of the matrix $A = \begin{bmatrix} -2 & 7 \\ 2 & 3 \end{bmatrix}$.

ii) State the condition for consistency of the system of linear equations $AX = B$.
Examine the following system of equations for consistency
$x + z = 2$
$-2x + y + 3z = 3$
$-3x + 2y + 7z = 4$

iii) Find the value of λ for which the system of equations have non - trivial solution.
$x + 2y + 3z = 0$
$2x + 3y + 4z = 0$
$3x + 4y + \lambda z = 0$

b) Verify Cayley Hamilton theorem for the matrix **2**

$$A = \begin{bmatrix} 1 & -5 \\ 3 & 2 \end{bmatrix}$$

5. a) i) Define orthogonal matrix **4**
prove that the matrix

$$A = \frac{1}{9}\begin{bmatrix} -8 & 4 & 1 \\ 1 & 4 & -8 \\ 4 & 7 & 4 \end{bmatrix}$$

is orthogonal.

ii) Reduce the quadratic form **4**
$x^2 - y^2 + z^2 - xy - 8yz + 2xz$
to its canonical form. Find its rank and index.

OR

a) i) For an orthogonal matrix A, show that $A^{-1} = A^T$. **4**
Verify whether the matrix A is orthogonal or not where

$$A = \frac{1}{3}\begin{bmatrix} 2 & 2 & -1 \\ 2 & -1 & 2 \\ -1 & 2 & 2 \end{bmatrix}$$

ii) Reduce the quadratic form **4**
$x^2 + 2y^2 + 2z^2 + 2xy - xz + 2yz$
to its canonical form. Find its rank, index and signature.

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