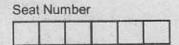April 2017

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MATHEMATICS PAPER - II : MTH-242
A) Differential Equations (241102) /
B) Differential and Difference Equations (241103)

P. Pages : 8

A) Differential Equations (241102)

Time : Two Hours   **Max. Marks : 60**

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicate full marks.

1.   a)   Attempt **any six** of the following.   6

   i)   The Wronskian of $\sin 3x$ and $\cos 3x$ is.
   a)   1   b)   3
   c)   -3   d)   None of these

   ii) Every continuous function ---------- satisfy a Lipschitz condition on a rectangle.
   a)   Must   b)   May
   c)   May not   d)   None of these

   iii) Two non zero solution $f_1(x)$ and $f_2(x)$ of the differential equation are linearly independent iff their Wronskian is ---------------.
   a)   Zero   b)   Non zero
   c)   Non vanishing   d)   None of these

   iv) The solution set of $\dfrac{dx}{z} = \dfrac{dy}{o} = \dfrac{dz}{-x}$ is
   a) $xz = c_1, y = c_2$ b) $z + x = c_1, y = c_2$
   c) $x^2 + z^2 = 2c_1, y = c_2$ d)   None of these

v) One set of multipliers of the differential equation

$$\frac{dx}{x(y^2+z)} = \frac{dy}{-y(x^2+z)} = \frac{dz}{z(x^2-y^2)}.$$

   a)   x, y, z   b)   x, -y, z

   c) $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ d)   None of these

vi) Which of the following equation is exact.

   a) $xy\,dy - y^2 dx$ b) $ye^x dx + e^x dy = 0$

   c) $x^2 y dx + y^2 dy = 0$ d)   None of these

vii) The equation Pdx + Qdy + Rdz = 0 is solve by method of auxiliary equation when.

   a)   Equation is not exact

   b)   Equation is integrable

   c)   Neither integrable nor exact

   d)   None of these

viii) $\overline{5/2} = $----------.

   a)   (5/2)!   b)   (3/2)!

   c) $\frac{3}{4}\sqrt{\pi}$ d) $\frac{15}{8}\sqrt{\pi}$

ix) $\int_0^\infty \frac{x^4}{(1+x)^9} dx = ---$

   a)   B(5, 4)   b)   B(4, 9)

   c)   B(5, 9)   d)   None of these

b) Attempt **any six** of the following.   6

   i)   Define linearly independent solution.

   ii)   Define Wronskian of $y_1(x)$, $y_2(x)$ and $y_3(x)$.

   iii) How many methods to find solution of differential equation

        $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$?

   iv) Find solution of the differential equation $\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$.

v) Define simultaneous differential equation of first order.

vi) Define homogeneous differential equation.

vii) State the condition of integrability of Pdx + Qdy + Rdz = 0.

viii) What is relation between Beta and Gamma function?

ix) $\int\limits_{0}^{\infty} \dfrac{x^{n-1}}{(1+x)^{m+n}} dx =$ ------------.

2. Attempt **any six** of the following. 12

i) Show that x and xe^x are linearly dependent on Y-axis.

ii) Find the Wronskian of $y_1 = e^x \sin x$ and $y_2 = e^x \cdot \cos x$.

iii) Solve $\dfrac{dx}{x^2 z} = \dfrac{dy}{0} = \dfrac{dz}{-x^2}$.

iv) Solve $\dfrac{dx}{a} = \dfrac{dy}{a} = dz$.

v) Show that $(2x + y^2 + 2xz) dx + 2xy\, dy + x^2\, dz = 0$ is integrable.

vi) Show that $(yz - x^3)dx + (zx - y^3)dy + (xy - z^3)dz = 0$ is exact.

vii) State the following statements are true or false.
   a) Every exact equation is integrable.
   b) Every integrable equation is exact.

viii) Prove that $\overline{|1} = 1$.

ix) Evaluate $\int\limits_{0}^{2} \dfrac{x^2}{\sqrt{2-x}} dx$.

3. Attempt **any four** of the following. 12

i) Two solution $y_1(x)$ and $y_2(x)$ of the equation
   $a_0(x)y'' + a_1(x) \cdot y' + a_2(x) \cdot y = 0$, $a_0(x) \ne 0 \, \forall x \in (a,b)$ are linearly
   dependent then prove that their Wronskian is identically zero.

ii) Solve $\dfrac{dx}{xy} = \dfrac{dy}{y^2} = \dfrac{dz}{zxy - 2x^2}$.

iii) Solve $\dfrac{dx}{z^2} = \dfrac{ydy}{xz^2} = \dfrac{dz}{xy}$.

iv) Solve $(yz + xyz)\, dx + (zx + xyz)\, dy + (xy + xyz)\, dz = 0$.

v) Evaluate $\displaystyle\int_0^1 (x \cdot \log x)^4\, dx$.

vi) Prove that $B(m, n) = B(m + 1, n) + B(m, n + 1)$.

4. Attempt **any three** of the following.   12

i) Show that $y_1 = e^x \sin x$ and $y_2 = e^x \cdot \cos x$ are linearly independent solution of differential equation $y'' - 2y' + 2y = 0$.

ii) Solve $\dfrac{dx}{x(2y^4 - z^4)} = \dfrac{dy}{y(z^4 - 2x^4)} = \dfrac{dz}{z(x^4 - y^4)}$.

iii) Solve $\dfrac{dx}{z} = \dfrac{dy}{-z} = \dfrac{dz}{z^2 + (x + y)^2}$.

iv) Solve $yz\, dx + 2zx\, dy - 3xy\, dz = 0$.

v) Prove that $\displaystyle\int_0^\infty \dfrac{x^{m-1}}{(a + bx)^{m+n}}\, dx = \dfrac{1}{a^n \cdot b^m} B(m,n)$.

5. Attempt **any two** of the following.   12

i) Solve $y'' + 4y = 4\tan 2x$ by the method of variation of parameter.

ii) Solve $(y^2 + yz)dx + (xz + z^2)dy + (y^2 - xy)dz = 0$ by the method of auxiliary equation.

iii) State and prove Duplication formula.

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