Oct-2013

Seat No.

MATHEMATICS PAPER-II MTH-232

A) : Topics in Algebra (New) (23116) OR / B) : Computational Algebra (New) (23117)

P. Pages : 4

A) : Topics in Algebra (New) (23116)

Time : Two Hours **Max. Marks : 40**

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Answersheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicate full marks.

1. Attempt **any eight** of the following. **8**

a) Define an abelian group.

b) Let $G=\{a^1,a^2,a^3......a^{11},a^{12}=e\}$ be a group. Find $O(a^9)$.

c) Define right cosets of subgroup H of group G.

d) Prepare additive composition table for $H=\{\overline{0},\overline{4},\overline{8}\}$ with modulo 12.

e) Define automorphism of group.

f) Let $F:G\to G'$ be a group homomorphism. If $x\in \ker f$ then $f(x)=.....$ for $x\in G$.

g) Define commutative ring.

h) Define integral domain.

i) Define division ring.

j) State Fermat's Theorem.

2. a) Attempt **any two** of the following. **6**

i) Let Q^+ denote the set of all positive rationals.

For $a,b\in Q^+$, define $a*b=\frac{ab}{2}$. Show that $\langle Q^+,*\rangle$ is abelian group.

ii) Let G be a group and $a^{-1}=a\ \forall a\in G$.

Prove that G is an abelian group.

iii) In $\langle Z_8', X_8\rangle$ find

a) $(\overline{3})^4$ b) $(\overline{3})^0$ c) $(\overline{3})^{-4}$

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b) Show that $\langle Z_5, +_5 \rangle$ is cyclic group. 2

3. Attempt **any two** of the following. 8

a) Prove that a nonempty subset H of group G is subgroup of G iff

$a, b \in H \Rightarrow ab^{-1} \in H$.

b) Find all subgroups of $\langle Z_{12}, +_{12} \rangle$

c) Find remainder when 3^{54} is divided by 11.

4. a) Attempt **any two** of the following. 6

i) Let $G = \langle Z, + \rangle$ be additive group of integers and

$G' = \{1, -1\}$ be a group under multiplication

Show that $f : G \to G'$ defined by

$$f(x) = \begin{cases} 1 \text{ if } x \text{ is even integer} \\ -1 \text{ if } x \text{ is odd integer} \end{cases}$$

is onto group homomorphism.

ii) Prove that the relation '$\cong$' of isomorphism of groups is an equivalence relation.

iii) Let $f : G \to G'$ be a group homomorphism prove that kerf is a subgroup of G.

b) Let $F : \langle Z, + \rangle \to \langle Z, + \rangle$ be isomorphism. Find kerf, when $f(x) = x \;\; \forall\, x \in Z$. 2

5. a) Define : 4

i) Commutative ring

ii) Division ring.

b) Define Boolean ring. Show that $\langle Z_2, \times_2 \rangle$ is Boolean ring. 4

OR

Let $\langle R, +, \cdot \rangle$ be a ring and $a, b, c \in R, 1 \in R$ 8

Prove that :

i) $a \cdot 0 = 0 \cdot a = 0$

ii) $a(-b) = -(ab) = (-a)b$

iii) $(-a)(-b) = ab$

iv) $(a - b)c = ac - bc$

v) $a(b - c) = ab - ac$

vi) $(-1) \cdot a = -a$

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B) : Computational Algebra (New) (23117)

Time : Two Hours **Max. Marks : 40**

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Answersheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicate full marks.
6. Use of calculator is not allowed.

1. Attempt **any eight** of the following. **8**
   i) Define a finite group.
   ii) In the group $(Z_6 +_6)$ find $O(\overline{4})$.
   iii) Define order of an element in the group.
   iv) State the Langranges theorem.
   v) Define Eulertotient function.
   vi) Consider (Z +) be the additive group of integers and $G=\{2^n / n \in Z\}$ be a group under multiplication. If we define $\phi : Z \to G$ as $\phi(n) = 2^n \ \forall \ n \in Z$ then show that ϕ is a homomorphism.
   vii) Define Kernel of homomorphism.
   viii) Define Hamming distance between two words.
   ix) Fill in the blanks.
   An (m, n) encoding function $e : B^m \to B^n \ m < n$ can detect K or fewer errors iff it's minimum distance is ------.
   x) Fill in the blanks.
   Let $e : B^m \to B^n$ is a group code then minimum distance of e is minimum weight of ------- codeword.

2. a) Attempt **any two** of the following. **6**
   i) Let Q^+ denotes the set of all positive rationals for $a, b \in Q^+$ define $a * b = \frac{ab}{3}$ then show that $(Q^+ *)$ is a group.
   ii) Show that the group G is abelian iff $(ab)^2 = a^2b^2 \ \forall \ a, b \in G$
   iii) Let G be a group and $a \in G$, $n \in Z$ then prove that $(a^n)^{-1} = (a^{-1})^n$.

   b) In a group G prove that every element of G has unique inverse. **2**

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3. Attempt **any two** of the following : 8

i) Let H and k be subgroups of a group G prove that $H \cup K$ is subgroup of G if and only if either $H \subseteq K$ or $K \subseteq H$.

ii) Prove that every subgroup of a cyclic group is cyclic.

iii) Find the remainder obtained when 33^{19} is divided by 7.

4. a) Attempt **any two** of the following : 6

i) Let G = (Z +) be the additive group of integers and $G' = \{1, -1\}$ is a group under multiplication show that $f : G \to G'$ defined by

$$f(n) = \begin{cases} 1 \text{ if n is even integer} \\ -1 \text{ if n is odd integer} \end{cases}$$

is a group homomorphism.

ii) Prove that every cyclic group of order n is isomorphic to $(Z_n +_n)$.

iii) Let $f : G \to G'$ be a group homomorphism then prove that f is one-one if and only if ker (f) = {e} where e is identity of G.

b) Prove that homomorphic image of an abelian group is abelian. 2

5. a) Let e be an (2, 5) encoding function defined by e (0 0) = 00000, e (1 0) = 10001, e (01) = 01110, e (11) = 11111. Show that e is a group code. How many errors will e detect. 4

b) Let $H = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be a parity check matrix decode the word 10100 relative to the maximum likelihood decoding function associated with eH. 4

OR

a) Let $H = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be a parity check matrix, determine the (2, 5) group code. 4

b) Let e be the (3, 5) encoding function defined as

$e(000) = 00000$ $e(100) = 01010$ $e(010) = 01101$

$e(110) = 00111$ $e(001) = 11110$ $e(101) = 10100$

$e(011) = 10011$ $e(111) = 11001$ and d is maximum likelihood decoding function associated with e then determine no. of errors that (e d) can correct. 4

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