April 2014

Seat No.

कड - 053

MATHEMATICS PAPER - II : MTH - 122
Algebra
(New) (12116)

P. Pages : 3

Time : Two Hours

Max. Marks : 40

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Answersheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to the right indicates full marks.
6. Use of calculator is not allowed.

1. Attempt **any eight** of the following. 8

i) State unique factorization theorem.

ii) State Fermat's theorem.

iii) Define Greatest common divisors.

iv) Define congruence relation modulo n.

v) If $Z_5 = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}, \overline{4}\}$ is the set of all residue classes of modulo 5. Find $\overline{3} X_5 \overline{4}$.

vi) Change the signs of the roots of the equation.
$x^5 + 7x^4 + 7x^3 - 8x^2 + x + 1 = 0$

vii) Find the equation whose roots are reciprocals of the roots of
$x^4 - 3x^3 + 7x^2 + 5x - 2 = 0$

viii) If p is prime and $a^2 \equiv b^2 \pmod p$, show that either $P / a + b$ or $P / a - b$.

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ix) If α, β, γ and δ are the roots of the equation $ax^4 + bx^3 + cx^2 + dx + e = 0$ then the value of $\sum\alpha\beta\gamma$ is ....

x) To remove the second term from the equation $x^4 - 4x^3 - 18x^2 - 3x + 2 = 0$, the roots are diminished by ....

2. a) Attempt **any two** of the following. 6

i) State and prove Euclid's lemma.

ii) Use the principle of finite induction, prove that $5^n + 3$ is divisible by $4\ \forall n \in N$.

iii) Show that $\sqrt{3}$ is not rational number.

b) Show that 4999 and 1109 are relatively prime. 2

3. Attempt **any two** of the following. 8

i) If $a \equiv b(\mathrm{mod}\, n)$ and $c \equiv d(\mathrm{mod}\, n)$, for a, b, c, d $\in$ z and $n \in N$ prove that

α) $(a+c) \equiv (b+d)(\mathrm{mod}\, n)$

β) $ac \equiv bd(\mathrm{mod}\, n)$

ii) A relation R defined in the set of integer z by $xRy \Leftrightarrow 7x - 3y$ is divisible by 4. Show that R is an equivalence relation in z.

iii) Find the remainder, when 15^{27} is divisible by 8.

4. a) Attempt **any two** of the following. 6

i) Find the condition that the roots of the equation $x^3 - px^2 + qx - r = 0$ are in A.P.

ii) Solve the equation $x^3 - 5x^2 - 2x + 24 = 0$ if the product of two of the roots is 12.

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iii) If α, β, γ are the roots of the equation $x^3 - px + qx - r = 0$ find the value of

$$\frac{1}{\alpha^2\beta^2} + \frac{1}{\alpha^2\gamma^2} + \frac{1}{\beta^2\gamma^2}.$$

b) If α, β, γ are the roots of the cubic equation $x^3 + px^2 + qx + r = 0$, Find $\sum\alpha^2\beta^2$. **2**

5. a) i) Explain Descarte's rule of signs for positive and negative roots. **4**

ii) Remove the second term from the equation $x^4 + 8x^3 + x - 5 = 0$ **4**

OR

a) i) Find the equation whose roots are those of $3x^3 - 2x^2 + x - 9 = 0$ each diminished by 5. **4**

ii) Solve the equation $x^3 - 21x - 344 = 0$ by Carden's method. **4**

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