Nov - 2015

Seat Number

MATHEMATICS PAPER - II : MTH - 232

(A) Topics in Algebra (23116) /

(B) Computational Algebra (23117)

P. Pages : 4

(A) Topics in Algebra (23116)

Time : Two Hours | Max. Marks : 40

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicate full marks.

1. Attempt **any eight** of the following : 8

   a) Define binary operation on G.

   b) Define cyclic group.

   c) Define Kernel of homomorphism.

   d) For what value of n, $\mathbb{Z}_n$ is an integral domain?

   e) What is order of an element '1' of additive group Z_6?

   f) State Euler's theorem.

   g) Let (IR, +) is group of real under addition and $(IR^+, \cdot)$ the group of positive reals under multiplication such that $f: IR \to IR^+$ defined by $f(x) = 2^x, \forall x \in IR$. find kerf.

   h) Define commutative ring.

   i) The generator of the multiplicative group $G = \{1, -1\}$ is -----.

   j) Is intersection of any two sub groups is subgroup?

2. a) Attempt **any two** of the following : 6

   i) Show that Q^+, the set of positive rationals is an abelian group under $a * b = \frac{a^b}{3}, \forall a, b \in Q^+$.

ii) Let G be a group and $a,b \in G$. Prove that the equation $y_a = b$ have unique solution in G.

iii) Prove that order of every element in a finite group is finite.

b) If G is a group prove that every element of G has unique inverse in G. 2

3. Attempt **any two** of the following : 8

i) Prove that every group of prime order is cyclic.

ii) Prove that a non empty subset H of a group G is subgroup of G if and only if $a,b \in H \Rightarrow ab^{-1} \in H$.

iii) Find the remainder obtained when 8^{401} is divided by 13.

4. a) Attempt **any two** of the following : 6

i) Prove that the homomorphic image of cyclic group is cyclic.

ii) Let $f: G \to G'$ be a group homomorphism, prove that, if H is subgroup of G then f(H) is subgroup of G'.

iii) Let G be a group and $f: G \to G$ be a map defined by $f(x) = x^{-1}$ $\forall x \in G$. Prove that, if G is abelian then f is an isomorphism.

b) Let $(\mathbb{Z},+)$ the group of integers under addition and $G = \{2n / n \in \mathbb{Z}\}$ a group under multiplication. Show that $f: \mathbb{Z} \to G$ defined by 2

$f(n) = 2^n$, $\forall n \in \mathbb{Z}$ is homomorphism.

5. a) Let $(R,+,.)$ be a ring and $a,b,c \in R$ prove that 8

i) $a.0 = 0.a = 0$

ii) $a(-b) = -ab = (-a) \cdot b$

iii) $(-a)(-b) = ab$

iv) $(a-b)c = ac - bc$.

OR

a) Prove that :

i) Every finite integral domain is a field. 4

ii) Every Boolean ring is a commutative ring. 4

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(B) Computational Algebra (23117)

Time : Two Hours **Max. Marks : 40**

Instructions to Candidates :
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2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicate full marks.
6. Use of calculator is not allowed.

1. Attempt **any eight** of the following : **8**

a) Obtain $\phi(7)$ where ϕ is Eulertotient function.

b) In the group $(Z_{12} +_{12})$ find $0(\overline{3})$.

c) Define subgroup.

d) State Lagranges theorem for the subgroup H of a group G.

e) Fill in the blanks : $3^{12} \equiv 1(\text{mod}......)$

f) Define Kernel of Homomorphism.

g) Let $(IR +)$be the additive group of reals and $(IR^{+} .)$ be multiplicative group of positive reals if $f : R \rightarrow IR^{+}$ defined as $f(x) = 10^{x}\ \forall x \in IR$ then show that f is a homomorphism.

h) Let $w_1 = 101011$ and $w_2 = 010100$the find hamming distance $\delta(w_1 w_2)$.

i) Choose the correct option
An (m, n) encoding function can detect k or fewer errors iff minimum distance minimum distance of e is ----- K+1.
i) at most ii) at least
iii) exactly iv) never

j) Define (n, m) decoding function.

2. Attempt **any four.** **8**

i) Let Q^{+} be the set of all positive rationals for $a, b \in Q^{+}$ define $a * b = \frac{ab}{10}$ then show that $(Q^{+} *)$ is a group.

ii) Let M_2 be the set of all 2×2 non zero matrices then check whether M_2 forms a group under multiplication or not. Justify your answer.

iii) Let G be a group and $a \in G$ be any element then show that $0(\overline{a}^{1}) = 0(a)$.

iv) Show that the group G is abelian iff $(ab)^2 = a^2b^2 \quad \forall a, b \in G$

v) Let G be a group, $a \in G$ and $n \in Z$ then prove that $(a^n)^{-1} = (a^{-1})^n$

vi) If G is a group such that $a^2 = e \quad \forall a \in G$ then show that G is abelian.

3. a) Attempt **any two.** 6

i) A nonempty subset H of a group G is subgroup of G iff $ab^{-1} \in H \quad \forall a,b \in H$

ii) Prove that every subgroup of a cyclic group is cyclic.

iii) Using Fermat's theorem find the remainder obtained when 33^{19} is divided by 7.

b) Prove that any two right cosets of H in G are either identical or disjoint. 2

4. Attempt **any two** 8

i) Let $G = (Z +)$ be the additive group of integers and $G' = \{1, -1\}$ be group under multiplication. If $f: Z \to G'$ is defined as

$$f(n) = \begin{cases} 1 \text{ if n is even} \\ -1 \text{ if n is odd} \end{cases}$$

then prove that f is onto homomorphism what is ker(f)?

ii) Let $f: G \to G'$ be a group homomorphism then prove that f is one-one iff ker(f) = {e}.

iii) Prove that every finite cyclic group of order n is isomorphic to $\langle Z_n \ +_n \rangle$.

5. a) Let $x,y,z \in B^m$ then show that 4

i) $\delta(x,y) = \delta(y,n)$ ii) $\delta(x,y) \geq 0$

iii) $\delta(x,y) = 0$ iff $x = y$ iv) $\delta(x,y) \leq \delta(x,z) + \delta(z,y)$

b) Consider (2 4) group code defined by e(0 0) = 0000 e(1 0)=1000 e(0 1) = 0111 e(11) = 1111 decode the following words relative to a maximum likelihood decoding function i) 0011 ii) 1011 iii) 1111 iv) 1001. 4

OR

a) Let $e: B^m \to B^n$ be a group code then prove that minimum distance of e is minimum weight of non zero code word. 4

b) Let $H = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be a parity check matrix decode the word 10100 relative to maximum likelihood decoding function associated with eH. 4

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