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## MATHEMATICS PAPER - II : MTH-242
# A) Topics in Differential Equations (24116) / B) Differential Equations and Numerical Methods (24117)

**P. Pages : 4**

*A) Topics in Differential Equations (24116)*

**Time : Two Hours** **Max. Marks : 40**

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Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicate full marks.

**1.** Attempt **any eight** of the following. **8**

a) Find the Wronskian of sin x and cos x.

b) If S is defined by the rectangle $|x| \le a, |y| \le b$ then the function $f(x, y) = x\sin y + y\cos x$ satisfy the Lipschitz condition. Here Lipschitz constant is ..........

i) a ii) -b iii) a+1 iv) b

c) Solve $dx = dy = \operatorname{cosec} x \, dz$

d) Solve dx = dy = dz.

e) Write the condition for the pfaffian differential equation Pdx + Qdy + Rdz = 0 is to be integrable.

f) Define pfaffian differential equation in n variables.

g) Define Beta function.

h) Find value of $v(6)$.

i) The value of integral $\int_0^1 x^3(1-x)^5 \, dx$ is ........

i) $\beta(4, 6)$ ii) $\beta(3, 5)$ iii) $\beta(3, 6)$ iv) $\beta(4, 5)$

j) If two non zero functions $F_1(x)$ and $F_2(x)$ are linearly dependent then write Wronskian of $F_1(x)$ and $F_2(x)$.

2. a) Attempt **any two** of the following. 6

i) By an example show that a continuous function may not satisfies Lipschitz condition.

ii) Using method of variation of parameters solve $y'' + y = \sec x$.

iii) Show that $\sin x$ and $\sin x - \cos x$ are linearly independent solutions of the differential equation $y'' + y = 0$.

b) Find Wronskian of $e^x \cos 2x$ and $e^x \sin 2x$. 2

3. Attempt **any two** of the following. 8

i) Solve $\frac{dx}{zy} = \frac{dy}{zx} = \frac{dz}{xy}$

ii) Solve $\frac{dx}{1} = \frac{dy}{3} = \frac{dz}{5z + \tan(y - 3x)}$

iii) Solve $\frac{dx}{y+z} = \frac{dy}{z+x} = \frac{dz}{x+y}$

4. a) Attempt **any two** of the following. 6

i) Solve $2yz\,dx + zx\,dy - xy(1+z)dz = 0$

ii) Solve $zy\,dx - zx\,dy - y^2 dz = 0$

iii) Solve $y(y+z)dx + x(x-z)dy + x(x+y)dz = 0$

b) Show that the equation $(x^2 - yz)dx + (y^2 - zx)dy + (z^2 - xy)dz = 0$ is exact. 2

5. a)

i) Evaluate $\int_0^\infty \frac{x^a}{a^x} dx$ 4

ii) Evaluate $\int_0^1 \sqrt{x}(1-x)\,dx$ 4

**OR**

i) Show that $\beta(m, n) = 2 \int_0^{\pi/2} (\sin\theta)^{2m-1} (\cos\theta)^{2n-1} d\theta$ 4

ii) Evaluate $\int_0^1 x^3 \log x\, dx$ 4

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## B) Differential Equations and Numerical Methods (24117)

**Time : Two Hours** **Max. Marks : 40**

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicate full marks.

**1.** Attempt **any eight** of the following. **8**

a) Define Wronskian of $y_1(x)$, $y_2(x)$ and $y_3(x)$.

b) Which of the following is solution of differential equation $y'' + 9y = 0$.

i) $e^{3x}$ ii) $e^{-3x}$ iii) $\sin 3x$ iv) $x^3$

c) Solve $\frac{dx}{y} = \frac{dy}{x} = \frac{dz}{xz}$

d) Fill in the blank $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R} = \frac{dx + 2dy - 3dz}{\text{-------}}$

e) State necessary and sufficient condition for integrability of the pfaffian differential equation Pdx + Qdy + Rdz = 0.

f) Show that the equation $(yz + 2x)dx + (zx - 2z)dy + (xy - 2y)dz = 0$ is exact.

g) Choose the correct option
Two solutions $y_1(x)$ and $y_2(x)$ of the equation $a_0(x)y'' + a_1(x)y' + a_2(x)y = 0$ $a_0(x) \neq 0 \ \forall \ x \in (a\ b)$ are linearly dependent iff their Wronskian is identically.

i) 1 ii) 0 iii) -1 iv) 2

h) Choose the correct option
Fourth order Runge Kutta formula is –

i) $y_{n+1} = y_n + \frac{1}{2}(k_1 + 2k_2 + 2k_3 + k_4)$ ii) $y_{n+1} = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

iii) $y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$ iv) $y_{n+1} = y_0 + \frac{1}{2}(k_1 + k_2 + k_3 + k_4)$

i) Write the modified Euler's formula to solve $\frac{dy}{dx} = f(x, y)$ $y(x_0) = y_0$

j) Fill in the blanks
Adams Bashforth predictor formula is $y_1 =$ --------.

2. a) Attempt **any two** of the following. 6

i) By an example, show that a continuous function may not satisfy the Lipschitz condition on a rectangle.

ii) Show that $y_1 = e^x \sin x$ and $y_2 = e^x \cos x$ are linearly independent solutions of differential equation $y'' - 2y' + 2y = 0$.

iii) Solve by the method of variation of para meters $y'' - 3y' + 2y = \frac{e^x}{1+e^x}$

b) Find the Wronskian of 1+x, $x^2$ and 1+2x. 2

3. Attempt **any two** of the following. 8

a) Solve $\frac{xdx}{y^2z} = \frac{dy}{zx} = \frac{dz}{yz}$

b) Solve $\frac{dx}{1} = \frac{dy}{3} = \frac{dz}{5z + \tan(y - 3x)}$

c) Solve by using method of Multipliers $\frac{dx}{y+z} = \frac{dy}{z+x} = \frac{dz}{x+y}$

4. a) Attempt **any two** of the following. 6

i) Explain the method of solving the homogeneous differential equation $Pdx + Qdy + Rdz = 0$ when $Px + Qy + Rz \neq 0$.

ii) Solve $zydx = zxdy + y^2dz$.

iii) Solve $xz^2dx - zdy + ydz = 0$ by using auxiliary equation.

b) Which of the following equations is homogeneous – 2

i) $(2xz - yz)dx + (2yz - xz)dy - (x^2 - xy + y^2)dz = 0$

ii) $x(x - y^2)dx + y(y - z^2)dy + z(z - x^2)dx = 0$

5. a) Solve the differential equation $\frac{dy}{dx} = 2x - y$ with $x_0 = 1, y_0 = 3$ by using Picard Method of successive approximations. Find up to third approximation. 4

b) Given $\frac{dy}{dx} = x + y, y(0) = 1$ find y for x = 0.05 correct to 4 decimal places by Modified Euler's method Take h = 0.05. 4

**OR**

a) Using Fourth order Runge Kutta method find y(0.1) given that $\frac{dy}{dx} = xy + y^2, y(0) = 1$ 4

b) Using Adams – Bashforth predictor – corrector method find y(0.4) given that $\frac{dy}{dx} = x^2 - y \;\; y(0.1) = 0.905125 \; y(0.2) = 0.8212352 \; y(0.3) = 0.7491509$. 4

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