Oct-2014

Seat Number

0528

MATHEMATICS PAPER - II : MTH-232
(A) Topics in Algebra (New) (23116) OR / (B) Computational Algebra (New) (23117)

P. Pages : 7

(A) Topics in Algebra (New) (23116) OR

Time : Two Hours **Max. Marks : 40**

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Answer sheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicate full marks.

1. Attempt any eight of the following. **8**

a) Which is identity element for addition in Z_n?

b) Let $G = \{a^1, a^2, a^3, ...a^{11}, a^{12} = e\}$ be a group find $O(a^3)$,

c) Define left coset of subgroup H of group G.

d) Prepare additive composition table for $H = \{\overline{0}, \overline{5}, \overline{10}\}$ with modulo 12.

e) Let $F : G \rightarrow G'$ be a group homomorphism. If $x \in \ker f$ then f(x) =.......... for $x \in G$.

f) Define group homomorphism.

g) Define field.

h) Define Boolean ring.

i) State Lagranges theorem for finite group.

j) Find generators of $G=\{1,-1, i, -i\}$ under multiplication of complex number.

2. a) Attempt **any two** of the following. 6

i) If G is group then prove that

1) Identity element of G is unique.

2) Every element of G has unique inverse in G.

ii) In the group $\langle z_7', x_7\rangle$ find

i) $(\bar{4})^{-3}$ ii) $0(\bar{4})$ iii) $(\bar{3})^2$

iii) Show that $Q = IR - \{-1\}$ is an abelian group under the binary operation $a * b = a + b + ab, \forall a, b \in G$.

b) Show that $\langle z_5, +_5\rangle$ is cyclic group. 2

3. Attempt **any two** of the following. 8

a) Show that $nZ = \{nr : r \in z\}$ is a subgroup of $\langle z, +\rangle$ where $n \in N$.

b) Let A, B be subgroups of a finite group G. Whose orders are relatively prime show that $A \cap B = \{e\}$.

c) Show that every proper subgroup of group of order 39 is cyclic.

4. a) Attempt **any two** of the following. 6

i) Consider $\langle IR, +\rangle$ a group of reals under usual addition. Show that $f: IR \to IR$ defined by $f(x) = 3x\ \forall x \in IR$ is group homomorphism. Find kerf.

ii) Prove that every infinite cyclic group is isomorphic to $\langle z, +\rangle$ the group of integers under addition.

iii) Let $f: G \to G'$ be a group homomorphism. If H' is a subgroup of G' then prove that $f^{-1}(H')$ is a subgroup of G.

b) Define auto morphism of groups. 2

5. Let $\langle R, +, \cdot \rangle$ be a ring and $a, b, c \in R$ prove that **8**

i) $a \cdot 0 = 0 \cdot a = 0$

ii) $a(-b) = -(ab) = (-a)b$

iii) $(-a)\,(-b) = ab$

iv) $a(b-c) = ab - ac$

v) $(a-b)c = ac - bc$

vi) $(-1)a = -a$ if $1 \in R$

OR

i) Prove that every field is an integral domain. **4**

ii) Prove that every Boolean ring is a commutative ring. **4**

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(B) Computational Algebra (New) (23117)

Time : Two Hours **Max. Marks : 40**

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Answer sheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to the right indicate full marks.
6. Use of calculator is not allowed.

1. Attempt **any eight** of the following. **8**

a) Define order of a group.

b) If ϕ is Euler's totient function then find $\phi(g)$.

c) In the group $(z +)$ of all integers explain whether the set of all odd integers is subgroup of $(z +)$ or not.

d) Let $H = \{\overline{0}, \overline{3}, \overline{6}, \overline{9}\}$ be the subgroup of $(z_{12} +_{12})$ then find the coset $H + \overline{4}$.

e) State Fermat's theorem.

f) Define an Isomorphism.

g) Consider $(z +)$ be the additive group of integers and $G = \{1, -1, i, -i\}$ be a group under multiplication. if we define $\phi : Z \rightarrow G$ as $\phi(n) = i^n \ \forall n \in z$ then find kernel of ϕ.

h) Define minimum distance of an encoding function $e: B^m \to B^n$ $m < n$.

i) Define a group code

j) Fill in the blanks
An (m, n) encoding function can detect k or fewer errors iff minimum distance of e is at least ....

2. a) Attempt **any two** of the following. 6

i) Show that $G = \mathbb{R} - \{1\}$ is an abelian group under the binary operation $a * b = a + b - ab \quad \forall a, b \in G$.

ii) If G is a group such that $a^2 = e \forall a, b \in G$ then show that G is abelian.

iii) In the group $(z_7' \ x_7)$ find

a) $(\bar{3})^2$ b) $(\bar{4})^{-3}$ c) $0(\bar{3})$

b) In any group G prove that the identity element is unique. 2

3. Attempt **any two** of the following. 8

i) Prove that every subgroup of a cyclic group is cyclic.

ii) Let H be the subgroup of a group G prove that $H_a = H_b$ iff $ab^{-1} \in H$.

iii) Find remainder obtained when 3^{54} is divided by 11.

4. a) Attempt **any two** of the following. 6

i) Let G be an abelian group and $f: G \to G$ be a map defined by $f(n) = x^{-1} \ \forall x \in G$ then prove that f is an isomorphism.

ii) Let $f: G \to G'$ be a group homomorphism then define kernel of f and show that ker (f) is subgroup of G.

iii) Prove that homomorphic image of a cyclic group is cyclic.

b) Consider $(\mathbb{R}\ +)$ be a group of reals under usual addition and $g: \mathbb{R} \to \mathbb{R}$ defined by $g(n) = x + 2 \quad \forall x \in \mathbb{R}$ then check whether g is a homomorphism or not. 2

5. a) Let e be an (3 5) encoding function defined by 4

$e(000) = 00000 \quad e(100) = 01010 \quad e(001) = 11110$

$e(101) = 10100 \quad e(010) = 01101 \quad e(110) = 00111$

$e(011) = 10011 \quad e(111) = 11001$

Show that e is a group code. How many errors will be detect.

b) Compute : 4

i) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \oplus \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

ii) $\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

OR

a) Let $H = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ 4

be a parity check matrix determine the (2, 5) group code.

b) Consider the (2,4) group code defined by e(00) = 0000 e(10) = 1000 e(01) = 0111 e(11) = 1111. Decode the following words relative to maximum likelihood decoding function. **4**

i) 0011 ii) 1011

iii) 1111 iv) 1001

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