Seat Number

April - 2016

कडबा - 012 / 013

MATHEMATICS PAPER - II : MTH-232
A) Topics in Algebra (23116) / B) Computational Algebra (23117)

P. Pages : 4

A) Topics in Algebra (23116)

Time : Two Hours **Max. Marks : 40**

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicate full marks.

1. Attempt **any eight** of the following. **8**
 a) Define order of an element in a group.
 b) Define subgroup.
 c) Define Homomorphism.
 d) For what value of P Z_p is field ?
 e) If P is prime then Euler's function $\phi(p)=\ldots\ldots$.
 f) State Lagranges theorem.
 g) If $f:(IR,+)\rightarrow(IR,+)$ defined by $f(x) = 2x$, $\forall x \in IR$ is group homomorphism. Find its Kernel.
 h) Define integral domain.
 i) In the multiplicative group $G=\{1,w,w^2\}$ of cube root of unity. The order of w is.....
 j) Give an example of ring which is not field.

2. a) Attempt **any two** of the following. 6

i) Let G be a group and $a, b \in G$. Prove that the equation ax=b have unique solution in G.

ii) Show that a group G is abelian if and only if $(ab)^2 = a^2 b^2 \; \forall a, b \in G$.

iii) If G is a group then prove that $(ab)^{-1} = b^{-1} \cdot a^{-1}, \forall a, b \in G$.

b) If in a group G every element is its own inverse then prove that G is abelian. 2

3. Attempt **any two** of the following. 8

i) Let A, B be subgroups of finite group G whose orders are relatively prime. Show that $A \cap B = \{e\}$.

ii) If a and b are two distinct element of a group G and H is subgroup of G then prove that Ha=Hb if and only if $ab^{-1} \in H$.

iii) State and prove Fermat's theorem.

4. a) Attempt **any two** of the following. 6

i) Let (Z, +) the additive group of integer and $G = \{1, -1, i, -i\}$ the group under multiplication. Show that $f: Z \to G$ defined by $f(n) = i^n, \forall n \in Z$ is a group homomorphism. Find its Kernel.

ii) Let G be a group and $a \in G$. Show that $f_a: G \to G$ defined by $f_a(x) = axa^{-1}, \forall x \in G$ is an isomorphism.

iii) Let $f: G \to G'$ be a group homomorphism. If H' is subgroup of G' then $f^{-1}(H')$ is subgroup of G.

b) Let $f: G \to G'$ be a group homomorphism prove that Ker f is subgroup of G. 2

5. a) On the set Z of integers, define binary operation $\oplus$ and $\odot$ as $a \oplus b = a + b - 1$ and $a \odot b = a + b - ab, \forall a, b \in Z$. Show that $(Z, \oplus, \odot)$ is a commutative ring with identity element o. 8

OR

a) i) Prove that a commutative ring $(R, +, \cdot)$ is an integral domain if and only if the cancellation law holds in R. 4

ii) Define field and Ring with zero divisor. 4

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