Seat Number

NOV-2016

कठोर - 023

* 1 0 5 3 *

MATHEMATICS PAPER - II : MTH - 112
Calculus of One Variable
(111102)

P. Pages : 4

Time : Two Hours **Max. Marks : 60**

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to the right indicates full marks.

1. A) Attempt **any six** of the following. **6**

i) $\lim_{x\to\infty} \frac{x}{e^x} =$

a) 1 b) 0
c) 2 d) none of these

ii) $\lim_{x\to 0} \left[\frac{2}{x^2-1} - \frac{1}{x-1}\right] =$

a) $-\frac{1}{2}$ b) $\frac{1}{2}$
c) 2 d) none of these

iii) Every differentiable function is

a) Discontinuous b) Continuous
c) Constant d) none of these

iv) For which value of $C \in (-1, 1)$ the Rolle's theorem is verified for the function $f(x)=x^2-1$ in $[-1, 1]$.

a) 1 b) -1
c) 0 d) none of these

v) If $y = x^n$ then $y_n =$

a) n! b) n
c) $(n-1)!$ d) none of these

vi) If $y = \sin(3x+5)$ then $y_5 =$

a) $243 \sin(3x+5)$ b) $243 \sin\left(3x+5+\frac{\pi}{2}\right)$

c) $243 \sin\left(3x+5+\frac{5\pi}{2}\right)$ d) None of these

vii) $\int_0^{\pi/2} \sin^9 x \, dx =$

a) $\frac{128}{315}$ b) $\frac{128\pi}{315}$

c) $\frac{63}{512}$ d) None of these

viii) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} +$ --------- is expansion of

a) e^x b) cosx

c) sinx d) None of these

B) Attempt **any six** of the following. 6

i) Define continuity of the function at x = a.

ii) Is every continuous function on closed and bounded interval is bounded?

iii) Define monotonic decreasing function.

iv) Find the value of C using L MVT for the function f(x) = logx in [i, e].

v) If $y = e^{2x}$ then y_6 = ----------.

vi) If $y = \log(ax+b)$ then y_n = ----------.

vii) Write the expansion of e^x.

viii) Write the reduction formula for $\int_0^{\pi/2} \cos^n x \, dx$.

2. Attempt **any six** of the following. 12

i) Evaluate $\lim_{x\to 0} \frac{\log \sin ax}{\log \sin bx}$.

ii) Discuss the continuity of $f(x) = \frac{x^2-4}{x-2}$, for $x \neq 2$
$= 4$, for $x = 2$ at point $x = 2$.

iii) If $f(x) = \frac{\log x - \log 5}{x-5}$, $x \neq 5$ is continuous at $x = 5$. Find f(5).

iv) Show that $f(x) = x^3 - 3x^2 + 3x + 2$ is monotonic increasing function of x in every interval.

v) In Lagrange's Mean Value theorem if $f(x) = 2x^2 - 7x + 10$, $x \in [2, 5]$ Find C.

vi) Find n^{th} derivative of $y = \sin^2 x$.

vii) If $y = \sin 5x \cdot \cos 3x$ find y_n.

viii) State Maclaurin's theorem for the function f(x).

ix) Evaluate $\int_0^{\pi/2} \sin^4 x \cdot \cos^6 x \, dx$.

3. Attempt **any four** of the following. 12

i) If $f(x) = \frac{(5^x - 1)^2}{\tan x \cdot \log (1+x)}$, for $x \neq 0$ is continuous at $x = 0$ find f(0).

ii) Discuss the continuity of the function
$f(x) = \frac{x^2}{2} - 2$, for $0 < x < 2$
$= 0$, for $x = 2$
$= 2 - \frac{8}{x^2}$, for $x > 2$ at point $x = 2$.

iii) Discuss the applicability of Rolle's theorem for the function $f(x) = (x-a)^m, (x-b)^n$ in [a, b], $m, n \in N$.

iv) Discuss the applicability of Cauchy's Mean Value theorem for the function $f(x) = x^2$, $g(x) = x^3$ in [1, 2].

v) If $y = x^3 e^x$ find y_n.

vi) Evaluate $\int_0^{\pi/6} \sin^6 3x \, dx$.

4. Attempt **any three** of the following. 12

i) Prove that continuous function on closed and bounded interval attains its bounds.

ii) If $f(x) = \frac{\sin 4x}{5x} + a$, for $x > 0$

$= x + 4 - b$, for $x < 0$

$= 1$, for $x = 0$.

is continuous at $x = 0$. Find a and b.

iii) Prove that $\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$, if $0 < a < b$.

Hence show that $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1} \frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$.

iv) If $y = a \cdot \cos(\log x) + b \sin(\log x)$ prove that

a) $x^2 y_2 + x y_1 + y = 0$.

b) $x^2 y_{n+2} + (2n+1) x y_{n+1} + (n^2+1) y_n = 0$

v) Using Taylor's theorem express the polynomial $f(x) = 2x^3 + 7x^2 + x - 6$ in power of $(x - 2)$.

5. Attempt **any two** of the following. 12

i) State and prove Lagrange's Mean Value theorem and find value of C, if $f(x) = 2x^2 - 10x + 29$ in [2, 7].

ii) Find n^{th} derivative of $y = \frac{1}{ax+b}$ and using it

find n^{th} derivative of $y = \frac{1}{4x+3}$.

iii) Prove reduction formula for $\int \frac{\sin nx}{\sin x} dx. (n > 1)$ and show that

$$\int \frac{\sin 6x}{\sin x} dx = 2\left[\frac{\sin 5x}{5} + \frac{\sin 3x}{3} + \sin x\right]$$

****