Seat Number

MATHEMATICS PAPER - II : MTH-232
A) Algebra (231102) /
B) Theory of Groups (231103)

P. Pages : 8

A) Algebra (231102)

Time : Two Hours **Max. Marks : 60**

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicate full marks.

1. A) Attempt **any six** of the following. 6

i) If G is a group then for any $a \in G$ $(a^{-1})^{-1}$ = ...

a) a^{-1} b) e
c) a d) None of these

ii) $(\mathbb{Z}_6, +_6)$ is a group with identify o then $o(\bar{4})$ = ------

a) 1 b) 2
c) 3 d) 4

iii) Let G be a group and $a \in G$. If $m, n \in \mathbb{N}$ such that o (a) = n and a^m= e then

a) $m \leq n$ b) $n \leq m$
c) n = m d) None of these

iv) Every subgroup of cyclic group is --------

a) Cyclic b) Not cyclic
c) Not subgroup d) None of these

v) Union of two subgroup of a group is ------

a) Subgroup b) Need not be subgroup
c) Cyclic d) None of these

vi) Homomorphic image of abelian group is ------

a) Cyclic
b) Not abelian
c) abelian
d) None of these.

vii) A one-one onto homomorphism is called --------

a) Isomorphism
b) Automorphism
c) Not isomorphism
d) None of these

viii) For P > 1 and $\mathbb{Z}_P$ is a field then P is -------

a) Composite
b) Prime
c) Even
d) None of these.

B) Attempt **any six** of the following. 6

i) Define binary operation.

ii) Define Euler's function $\phi(n)$

iii) State Lagrange's theorem.

iv) Define right coset

v) Define homomorphism.

vi) If $f : \mathbb{C}^* \to \mathbb{R}^*$ defined by $f(z) = |z|$, $\forall z \in \mathbb{C}^*$ is a group homomorphism under multiplication then $f(z_1, z_2)$ = ---------- $\forall z_1, z_2 \in \mathbb{C}^*$

vii) Define commutative ring

viii) Define field.

2. Attempt **any six** of the following. 12

i) If in a group G. every element is its own inverse then prove that G is abelian.

ii) Let G be a group and a, b, c, ∈ G prove that ab = ac then b = c.

iii) Let G = {1, -1, i, -i} is a group w. r. t. usual multiplication find order of each element of G.

iv) If G is abelian group prove that every k/f coset is equal to right coset.

v) Prove that order of every element of a finite group is divisor of order of group.

vi) If (IR, +) is a group of reals under addition and f: IR→IR defined by f(x) = 2x is homomorphism. Find kernel of f.

vii) If a commutative ring (R, +, ·) is an integral domain then prove that the cancellation law hold in R.

viii) If a ring R is commutative then prove that $(a+b)^2 = a^2+b^2+2ab$, $\forall$ a, b $\in$ R.

ix) Let H be a subgroup of group G and a, b $\in$ G such that Ha = Hb prove that $ab^{-1} \in H$.

3. Attempt **any four** of the following. **12**

i) Let G be a group and a, b $\in$ G Prove that the equation ya = b have unique solution in G.

ii) Prove that a group having four element must be abelian.

iii) Prove that intersection of any two subgroup is a subgroup.

iv) Prove that every group of prime order is cyclic.

v) Let G be a group and a $\in$ G. Show that $f_a : G \to G$ defined by $f_a(x) = axa^{-1}$ $\forall$ x $\in$ G is an isomorphism.

vi) Prove that every Boolean ring is commutative.

4. Attempt **any three** of the following. **12**

i) Let G be a group and a$\in$ G show that o (a) = o (a^{-1})

ii) find the remainder obtained when 15^{27} is divided by 8.

iii) Let $G = \{a, a^2, a^3, ----, a^{11}, a^{12} = e\}$ be a cyclic group of order 12 generated by a -show that $f : G \to G$ defined by $f(x) = x^4$, $\forall$ x $\in$ G is group homomorphism and find its kernel.

iv) Prove that every finite integral domain is field.

v) Show that $\mathbb{Z}$ be an abelian group under $a * b = a + b + 1, \forall a, b \in \mathbb{Z}$.

5. Attempt **any two** of the following. 12

i) Define subgroup and prove that a nonempty subset H of a group G is subgroup if and only if $a b^{-1} \in H, \forall a, b \in H$.

ii) Let $f : G \to G'$ be a group homomorphism then prove that.

a) If e is identity element of G then f(e) is identity element of G'

b) $f(a^{-1}) = [f(a)]^{-1}, \forall a \in G$

c) $f(a^m) = [f(a)]^m, \forall a \in G$ and $m \in \mathbb{Z}$.

iii) Denote $R = 2\mathbb{Z}$ = the set of even integers for a, b$\in$ R we define a+b = usual addition of a and b and $a \odot b = \frac{ab}{2}$. Show that $(R, +, \odot)$ is commutative ring with identity element 2.

****