Oct-2013

Seat No. | | | | | | |

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केंद्रक - 039

MATHEMATICS PAPER - II (NEW) (12116) MTH-122

Algebra

P. Pages : 2

Time : Two Hours

Max. Marks : 40

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Answersheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to the right indicate full marks.
6. Use of calculator is not allowed.

1. Attempt **any eight** of the following. 8

i) State division algorithm theorem.

ii) Define relatively prime integers.

iii) Define Equivalence relation.

iv) State Euler's theorem.

v) If P is prime and $a^2 \equiv b^2 \pmod{p}$. Show that either $P/a+b$ or $P/a-b$.

vi) State Descarte's rule of sign for positive root.

vii) Change the signs of the roots of the equation $x^7 + 7x^6 + 3x^4 - 4x^2 - 3x + 2 = 0$.

viii) Find the equation whose roots are the reciprocals of the roots of $x^4 - 5x^3 + 7x^2 + 3x - 7 = 0$.

Fill in the blanks of the following.

ix) If α, β, γ are the roots of the equation $ax^3 + bx^2 + cx + d = 0$ then $\sum\alpha =$ ..... and $\sum\alpha\beta =$ .....

x) To remove the second term from the equation $x^4 + 20x^3 + 143x^2 + 430x + 462 = 0$, the roots are diminished by .....

2. a) Attempt **any two** of the following. 6

i) If p is prime and a, b are integers such that p/ab, Prove that either p/a or p/b

केंद्रक - 039

ii) Find $(27 > 2,\ 2 > 3)$ and express it in the form $27 > 2m + 2 > 3n$ where m, n are integers.

iii) Show that $\sqrt{5}$ is not rational number.

b) If a/b and b/c, Prove that a/c where $a,b,c \in Z$. **2**

3. Attempt **any two** of the following. **8**

i) State and prove Fermat's theorem.

ii) Prepare the composition table for addition and multiplication of residue classes modulo 7.

iii) Let A be the set of all integers. Define the relation R on A by xR_y if "$x - y$ is an even integer" $x, y \in A$. Prove that R is an equivalence relation.

4. a) Attempt **any two** of the following. **6**

i) If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$. Find the value of $(\beta + \gamma)(\gamma + \alpha)(\alpha + \beta)$.

ii) Solve the equation $x^3 - 3x^2 + 4 = 0$ if two its roots are equal.

iii) Find the condition that the cubic equation $x^3 + px^2 + qx + r = 0$. Should have two roots α, β connected by the relation $\alpha\beta + 1 = 0$.

b) If α, β, γ are the roots of the cubic equation $x^3 + px^2 + qx + r = 0$, Find $\sum\alpha^2$. **2**

5. a) i) Reduce the cubic equation $2x^3 - 2x^2 + 6x - 1 = 0$, to the form $Z^3 + 3HZ + G = 0$. **4**

ii) Solve the equation $x^3 - 21x - 344 = 0$ by Carden's method. **4**

OR

a) i) Remove the fractional coefficients from the equation $x^3 - \frac{5}{2}x^2 - \frac{7}{18}x + \frac{1}{108} = 0$. **4**

ii) Find the equation whose roots are the roots of $x^4 - 5x^3 + 7x^2 - 17x + 11 = 0$ each diminished by 4. **4**

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