March 2016/7

Seat Number

गज - 001

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MATHEMATICS PAPER - I : MTH-111
Matrices
(111101)

P. Pages : 4

Time : Two Hours **Max. Marks : 60**

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicates full marks.

1. a) Attempt **any six** of the following. **6**

i) If A is a square matrix and $|A| \neq 0$ then $A^{-1} =$ ------------

a) $\frac{1}{|A|}(\text{adj}A)$ b) $|A|\frac{1}{\text{adj}A}$

c) $|A| \cdot (\text{adj } A)$ d) None of these

ii) If A is non singular matrix of order n then adj A is ---------- matrix.

a) not square b) singular

c) non-singular d) None of these

iii) If every minor of matrix A of order 7 is zero then $\rho(A) <$ -----.

a) 0 b) 7

c) 6 d) None of these

iv) The inverse of elementary matrix $E_{\xi}(k)$ is---------

a) $E_{\xi}(-k)$ b) $E_{\xi}\left(\frac{1}{k}\right)$

c) $E'_{\xi}(k)$ d) None of these

v) If $\rho(A) = \rho(AB) = n$, the number of unknows then $Ax = B$ possesses a ----------- solution.

a) Infinite b) Unique

c) Trivial d) None of these

vi) If λ is non zero eigen value of a non singular matrix A then eigen value of A^{-1} is ----------

a) $\frac{1}{\lambda}$ b) $-\frac{1}{\lambda}$

c) λ d) None of these

vii) An orthogonal matrix A is called proper orthogonal if $|A|$ = ------------.

a) 1 b) 0

c) -1 d) None of these

viii) A quadratic form is said to be negative definite if ---------

a) $r = n, s = o$ b) $r = s = n$

c) $r < n, s = o$ d) None of these

b) Attempt **any six** of the following. 6

i) If $A = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 2 & 1 \\ 3 & 1 & 0 \end{bmatrix}$ find minor of a_{11} and a_{23}.

ii) Define cofactor of an element of a matrix.

iii) What is the inverse of elementary matrix $E_{32}(-5)$.

iv) Define equivalent matrices.

v) Define eigen value of corresponding matrix A.

vi) State the condition that the non-homogeneous system Ax = B is consistent.

vii) Find the matrix of quadratic form $Q = x_1^2 + 6x_1x_2 - 12x_1x_3 + 4x_3^2$

viii) Prove that the matrix $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

2. Attempt **any six** of the following. 12

i) If A is non singular matrix then prove that $\left(A^1\right)^{-1} = \left(A^{-1}\right)^1$.

ii) If A and B are symmetric matrices then prove that (AB + BA) is symmetric matrix.

iii) Define rank of matrix.

iv) If $I=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ then compute $E_{12}\cdot E'_{13}(-1)$

v) Define Linearly dependent & Linearly independent solution of system of $Ax=0$.

vi) Find eigen value of matrix $A=\begin{bmatrix}3&-5\\7&8\end{bmatrix}$

vii) Show that the matrix $A=\begin{bmatrix}\cos\theta&-\sin\theta\\\sin\theta&\cos\theta\end{bmatrix}$ is proper orthogonal.

viii) Define canonical form of quadratic form.

ix) Prove that Inverse of matrix if it exist is unique.

3. Attempt **any four** of the following. **12**

i) Let $A=\begin{bmatrix}-3&1&0\\2&-2&1\\-1&-1&1\end{bmatrix}$ show that $A\cdot(\text{adj }A)$ is null matrix.

ii) Verify that $(AB)^{-1}=B^{-1},A^{-1}$, where $A=\begin{bmatrix}3&1\\1&1\end{bmatrix}$, $B=\begin{bmatrix}3&-1\\2&1\end{bmatrix}$

iii) Reduce the matrix $A=\begin{bmatrix}1&2\\-2&-4\\3&6\end{bmatrix}$ to its normal form hence find $\rho(A)$

iv) Determine the value of x so that the matrix $A=\begin{bmatrix}x&x&1\\1&x&x\\x&1&x\end{bmatrix}$ is of rank 3.

v) Investigate for what values of λ and μ, the system of equation $x+3y+2z=2,\ 2x+7y+3z=-11,\ x+y+\lambda z=\mu$ have no solution.

vi) If A is an orthogonal matrix then prove that $A^1=A^{-1}$.

4. Attempt **any three** of the following. 12

i) If $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ then find A^{-1}.

ii) If $A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 0 \\ 1 & 2 \end{bmatrix}$ verify that $\operatorname{adj}(AB) = (\operatorname{adj} B) \cdot (\operatorname{adj} A)$.

iii) Examine for nontrivial solutions $x + y + z = 0$, $4x + y = 0$, $2x + 2y + 3z = 0$

iv) Find the rank of a matrix A where

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

v) Write down the quadratic form corresponding to the matrix.

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 3 & -2 \\ 3 & -2 & 4 \end{bmatrix}$$

5. Attempt **any two**. 12

i) If A is matrix of rank r then prove that there exist two nonsingular matrices P and Q such that $PAQ = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$

ii) Verify Cayley Hamiton theory for $A = \begin{bmatrix} 1 & 2 & 0 \\ -3 & -2 & 1 \\ 1 & 3 & -1 \end{bmatrix}$

iii) Obtain the linear transformation of the quadratic form $q = x_1^2 - x_2^2 + x_3^2 - 2x_1x_2 + 4x_2x_3$ under the linear transformation.

$x_1 = y_1 + y_2 + y_3$

$x_2 = y_2 - y_1$

$x_3 = 2y_3$

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