Nov-2015

Seat Number

MATHEMATICS PAPER - I (NEW) : MTH- 241

Complex Analysis

(24115)

P. Pages : 3

Time : Two Hours Max. Marks : 40

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicate full marks.

1. Attempt **any eight** of the following. **8**

a) Find the modulus of $\frac{1}{1+i}$

b) Find the value of $\left|\frac{z-1}{1-\bar{z}}\right|$

c) Evaluate $\lim_{z\to 0}\frac{\bar{z}}{z}$ along real axis.

d) Define $f'(z_0)$.

e) State Cauchy Riemann equations.

f) State the series for sinz.

g) State Cauchy Goursat theorem.

h) Find the poles of $f(z)=\frac{z-1}{(z^2-3z+2)(z-3)}$ in complex plane.

i) Find the zeros of $f(z)=z^2-5z+6$.

j) If $z = e^{i\theta}$ then find the value of $\cos\theta$ in terms of z.

2. a) Attempt **any two** of the following. 6

i) Find the n – n^{th} roots of z = x + iy.

ii) Simplify $\frac{(\cos 2\theta - i\sin 2\theta)^7 (\cos 3\theta + i\sin 3\theta)^{-5}}{(\cos 4\theta - i\sin 4\theta)^{-12} (\cos 5\theta - i\sin 5\theta)^{-6}}$

iii) If $|z_1| = |z_2| = |z_3| = 5$ and $z_1 + z_2 + z_3 = 0$ then prove that $\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} = 0$

b) Solve $x^2 - 1 = 0$. 2

3. Attempt **any two** of the following. 8

a) If $w = f(z) = u(x, y) + iv(x, y)$ be an analytic function at a point z = x + iy and the partial derivatives of u and v exist then prove that $f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$

b) Discuss the continuity of the function

$$f(z) = \frac{z^2 + 4}{z - 2i} \text{ if } z \neq 2i$$
$$= 3 + 4i \text{ if } z = 2i$$

at z = 2i.

c) Find the analytic function $f(3) = u = iv$ whose imaginary part is $e^x(x\sin y + y\cos y)$.

4. a) Attempt **any two** of the following. 6

i) State Cauchy's integral formula for $f(z_0)$ and hence obtain $\int_C \frac{z^3}{z - 2i} dz$ where C is the circle $|z - 2| = 5$.

ii) Using Cauchy Goursat theorem obtain the value of $\int_C e^z dz$ where C is the circle $|z| = 1$ and hence deduce that $\int_0^{2\pi} e^{\cos\theta} \cdot \cos(\theta + i\sin\theta)d\theta = 0$

iii) Evaluate $\int_{|z-1|=z} \frac{\sin \pi z}{(z-1)^2} dz$

b) Write the expansion of $(1-z)^{-1}$ for $|z| < 1$. **2**

5. a) State and prove Cauchy's residue theorem. **4**

b) Compute the residue at double pole of $f(3) = \frac{z^2 + 2z + 3}{(z-i)^2(z+4)}$ **4**

OR

a) Find the residue of $\frac{ze^z}{(z-1)^3}$ at its pole. **4**

b) Use contour integration evaluate $\int_0^{2\pi} \frac{d\theta}{5 + 3\cos\theta}$ **4**

****