Nov-2016

Seat Number

0637

काठ - 036 / 037

MATHEMATICS PAPER - II : MTH - 232
A) Algebra (231102) / B) Theory of Groups (231103)

P. Pages : 8

A) Algebra (231102)

Time : Two Hours **Max. Marks : 60**

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicate full marks.

1. a) Attempt **any six** of the following. **6**

i) If G is a group and $a, b \in G$ then $(ab)^{-1}$=........
   a) $a^{-1}b^{-1}$ b) ab
   c) $b^{-1}a^{-1}$ d) None of these

ii) If $\phi(n)$ is Euler's function then $\phi(10)$ = ...
   a) 3 b) 4
   c) 5 d) None of these

iii) Any two left coset of H in group G are -----
   a) Identical b) Not identical
   c) Not equal d) None of these

iv) Every proper subgroup of a group of order 55 are ------
   a) Not cyclic b) Cyclic
   c) Not subgroup d) None of these

v) A isomorphism from a group G onto itself is called -----
   a) Homomorphism b) Isomorphism
   c) Auto morphism d) None of these

vi) Every finite cyclic group of order n is isomorphic to

a) $(\mathbb{Z},+)$ b) $(\mathbb{Z}_n,+_n)$

c) (IR, +) d) None of these

vii) If P is prime then $\mathbb{Z}_p$ is ------

a) Not ring b) Boolean ring

c) Finite field d) None of these

viii) If R is commutative ring and $a, b \in R$ then $(a+b)^2 = ...$

a) $a+b$ b) a^2+b^2+2ab

c) $a^2+b^2+ab+ba$ d) None of these

b) Attempt **any six** of the following. 6

i) Define order of an element in a group.

ii) Define group.

iii) State Fermat's theorem.

iv) Is every cyclic group is abelian?

v) Define Dihedral group.

vi) Define Kernel of homomorphism.

vii) Let $\mathbb{Z}$ be the additive group of integer and $f:\mathbb{Z}\to\mathbb{Z}$ define by $f(n)=2n$ then $f(m+n)=.......$

viii) Define integral domain.

2. Attempt **any six** of the following. 12

i) Show that $G=\{1,-1,i,-i\}$ is a group w.r.t. usual multiplication of complex number.

ii) If G is a group such that $a^2=e, \forall a \in G$ show that G is abelian.

iii) Let $(\mathbb{Z}_6,+_6)$ is a group with identity 0. Find order each element of $\mathbb{Z}_6$.

iv) If a is an element of a finite group G then prove that $a^{0(G)}=e$.

v) Let H be a subgroup of group G and $a \in G$ then show that

$Ha = \{x \in G / xa^{-1} \in H\}$

vi) If $f: G \to G'$ be a group homomorphism prove that if e is the identity element of G then $f(e)$ is identity element of G'.

vii) G be a group of non zero real number under multiplication and $f: G \to G$ define by $f(x) = x^2$ is homomorphism. Find kernel of f.

viii) Define ring.

ix) Prove that $(\mathbb{Z}, +, \cdot)$ is an integral domain but not field.

3. Attempt **any four** of the following. 12

i) Let G be a group and $a, b \in G$ prove that the equation $ax = b$ have unique solution in G.

ii) Show that a group G is abelian if and only if $(ab)^2 = a^2 b^2, \forall a, b \in G$.

iii) Let H be a subgroup of a group G. Prove that $a \in H$ if and only if Ha = H.

iv) Prove that every proper subgroup of a group of order 77 is cyclic.

v) Let $f: G \to G'$ be a group homomorphism. Prove that if H is subgroup of G then f (H) is subgroup of G'.

vi) Prove that every field is integral domain.

4. Attempt **any three** of the following. 12

i) Show that Q^+ the set of all positive rationals is an abelian group under $a * b = \frac{ab}{2} \forall a, b \in Q^+$.

ii) Let G be a group and $a \in G, n \in N$ show that $a^n = e$ if and only if $0(a)/n$.

iii) Find the remainder when 6^{412} is divided by 13.

iv) Let $f: G \to G'$ be a group homomorphism. Prove that f is one - one if and only if $\ker f = \{e\}$ where e is identity element of G.

v) Let R be a ring with identity element '1' and $(ab)^2 = a^2 b^2, \forall a, b \in R$ show that R is commutative.

5. Attempt any two of the following. 12

i) State and prove Lagrange's theorem for finite group.

ii) Let G be a group and $f: G \to G$ be a map defined by $f(x) = x^{-1}, \forall x \in G$

Prove that

a) If G is abelian then f is an isomorphism.

b) If f is a group homomorphism then G is abelian.

iii) On the set $\mathbb{Z}$ of integers define binary operation $\oplus$ and $\odot$ as $a \oplus b = a + b - 1$ and $a \odot b = a + b - ab \; \forall \; a, b \in \mathbb{Z}$ show that $(\mathbb{Z}, \oplus, \odot)$ is a ring.

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