oct-2013

Seat No.

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MATHEMATICS PAPER-I (NEW) (23115) MTH-231

Advanced Calculus

P. Pages : 4

Time : Two Hours

Max. Marks : 40

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Answersheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicates full marks.

1. Attempt **any eight** of the following. 8

a) Let $f(x,y)=\dfrac{x^2y^2}{x^4+y^4-x^2y^2}(x,y)\neq(0,0)$

verify that both the repeated limits exist and are equal.

b) If $u=x^3+y^4,\ x=t^2,\ y=t$ then find $\dfrac{du}{dt}$ at $t=1$.

c) Write the condition for critical point (a, b) to become a function f (x, y) minimum.

d) Evaluate $\int_0^1\int_0^1 xy\ dx\,dy$

e) If $f(x,y)=\dfrac{xy}{x^2+y^2},\ (x,\ y)\neq(0,0)$.

and f (0, 0) = 0 then fx (0, 0) is ....

f) If $u=x^3z+xy^2-2yz$ then $\dfrac{\partial u}{\partial x}$

at the point (1, 2, 3) is ....

g) Stationary points of the function u (x, y) are obtained by
   i) ux = 0
   ii) uy = 0
   iii) ux = 0 and uy = 0
   iv) None of these.

h) Degree of homogeneous function

$$f(x,y)=\left[\frac{x^{\frac{1}{2}}+y^{\frac{1}{2}}}{x^{\frac{1}{3}}+y^{\frac{1}{3}}}\right]^{\frac{1}{2}} \text{ is ....}$$

   i) 2 ii) $\frac{1}{3}$ iii) $\frac{1}{12}$ iv) None of these.

i) Define Saddle point.

j) Define Triple integral.

2. a) Attempt **any two** of the following. 6

   i) Discuss the continuity of the function f (x, y) at (0, 0)

$$f(x,y)=\frac{xy}{\sqrt{x^2+y^2}} \text{ when } x\neq 0,\ y\neq 0 \text{ and } f(0,0)=0.$$

   ii) If $u=x^2\tan^{-1}\frac{y}{x}-y^2\tan^{-1}\frac{x}{y}$

   prove that $\frac{\partial^2 u}{\partial x\partial y}=\frac{x^2-y^2}{x^2+y^2}$

   iii) Using differentials, find approximate value of $\sqrt{(1.02)^2+(1.97)^3}$.

b) If $u=\log(\tan x+\tan y+\tan z)$ then prove that 2

$$\sin 2x\frac{\partial u}{\partial x}+\sin 2y\frac{\partial u}{\partial y}+\sin 2z\frac{\partial u}{\partial z}=2$$

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3. Attempt **any two** of the following. 8

i) If u = f (x, y) is a differentiable function of x and y and $x = \phi(t)$ and $y = \psi(t)$ are differentiable functions of t then the composite function $u = f[\phi(t), \psi(t)]$ is a differentiable function of t and

$$\frac{du}{dt} = \frac{\partial u}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial u}{\partial y} \cdot \frac{dy}{dt}$$

ii) If $f(x, y) = x^3 - xy^2$ show that θ used in the mean value theorem applied to the points (2, 1) and (4, 1) satisfies the equation

$$3\theta^2 + 6\theta - 4 = 0$$

iii) If $u = \tan^{-1}\left[\frac{x^3 + y^3}{x - y}\right]$ then prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$$

4. a) Attempt **any two** of the following : 6

i) If f (x, y) is defined on the region R such that f possesses continuous partial derivatives of order n in a certain neighbourhood of point (a, b) such that

1) fx (a, b) = fy (a, b) = 0

2) $fxx(a, b) - fyy(a, b) - (fxy(a, b))^2 > 0$

3) $fxx(a, b) < 0$ then prove that f has maxima at (a, b).

ii) Prove that

$$\sin(x + y) = (x + y) - \frac{(x + y)^3}{3!} + \ldots\ldots$$

iii) Find the rectangle of perimeter 12cm which has maximum area.

b) State Taylors theorem for functions of two variables. 2