April-2016

Seat Number

0518

MATHEMATICS PAPER - II : MTH-112
Calculus of One Variable
(111102)

P. Pages : 4

Time : Two Hours

Max. Marks : 60

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All question are compulsory.
5. Figures to the right indicates full marks.

1. a) Attempt **any six** of the following. **6**

i) $\lim_{x \to 0} \frac{x - \tan x}{x^3}$

a) $\frac{1}{3}$ b) $-\frac{1}{3}$

c) 3 d) -3

ii) $\lim_{x \to 0} (1+4x)^{\frac{1}{x}}$

a) e^4 b) e

c) 4 d) $e^{\frac{1}{4}}$

iii) For which value of C E (1, 5) the Rolle's theorem is verified for the function $f(x) = x^2 - 6x + 5$ in [1, 5].

a) 2 b) -3

c) 3 d) none of these

iv) If $f(x) = |x|$ then $f(x)$ is

a) continuous but not differentiable at x = 0.

b) continuous and differentiable at x = 0.

c) differentiable but not continuous at x = 0.

d) None of these.

v) If $y = e^{ax}$ then $y_n =$

a) e^{ax}

b) $a^n . e^{ax}$

c) $a . e^{ax}$

d) none of these

vi) If $y = \cos(ax+b)$ then $y_n =$

a) $a^n . \cos\left(ax + b + n\frac{\pi}{2}\right)$

b) $a^n . \sin\left(ax + b + n\frac{\pi}{2}\right)$

c) $\sin(ax+b)$

d) None of these

vii) $\int_0^{\pi/2} \cos^{10} x \, dx$

a) $\frac{63}{512}$

b) $\frac{63\pi}{512}$

c) $\frac{63\pi}{315}$

d) None of these

viii) $1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$ is expansion of

a) $\sin x$

b) e^x

c) $\cos x$

d) $\log(1+x)$

b) Attempt **any six** of the following. 6

i) Is every bounded function in the closed interval is continuous?

ii) If $f(x)$ is continuous in [a, b] and $f(a) \neq f(b)$ then $f(x)$ assume every value between -----

iii) Define derivative of a function at a point.

iv) Show that $f(x) = x^3 + 3x + 2$ is monotonic increasing function in every interval.

v) If $y = x^5$ then $y_6 = \ldots\ldots$

vi) If $y = \frac{1}{2x+5}$ then $y_n = \ldots\ldots$

vii) Write the expansion of cos x.

viii) Write reduction formula for $\int_0^{\pi/2} \sin^n x \, dx$

2. Attempt **any six** of the following. 12

i) Evaluate $\lim_{x \to 0} \frac{e^x - 1 - \log(1+x)}{x^2}$

ii) Discuss the continuity of $f(x) = \frac{x^2-9}{x-3}$, for $x \neq 3$
$= 5$, for $x = 3$ at point $x = 3$

iii) If $f(x) = (1+3x)^{\frac{1}{x}}$, $x \neq 0$ is continuous at $x = 0$, find $f(0)$.

iv) In LMVT if the function $f(x) = x^2 - 3x + 2$ in [-2, 3] find C.

v) State Rolle's theorem.

vi) If $y = e^{3x}.\cos 4x$ then $y_n = \ldots..$

vii) Find n^{th} derivative of $y = \frac{1}{(x+2)(x+3)}$

viii) State the reduction formula for $\int \cos^n x \, dx$.

ix) Evaluate $\int_0^{\pi/2} \sin^6 x . \cos^3 x \, dx$.

3. Attempt **any four** of the following. 12

i) Discuss the continuity of the function
$f(x) = x.\sin\frac{1}{x}$, for $x \neq 0$
$= 0$, for $x = 0$, at point $x = 0$.

ii) If $f(x) = \frac{(4^{\tan x} - 1)^2}{x.\log(1+x)}$, for $x \neq 0$ is continuous at $x = 0$. find $f(0)$.

iii) Discuss the applicability of Rolle's theorem for the function
$f(x) = e^x(\sin x - \cos x)$ in $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$.

iv) Prove that every differentiable function is continuous.

v) If $y = x^2.\sin(3x+7)$ find y_8.

vi) Evaluate $\int_0^{\infty} \frac{x^6}{(1+x^2)^{11/2}} dx$

4. Attempt **any three** of the following. **12**

i) Prove that every continuous function on closed and bounded interval is bounded.

ii) If $f(x)$ is continuous in [0, 8] where

$$\begin{aligned} f(x) &= x^2 + ax + b, && \text{for } 0 \le x < 2 \\ &= 3x + 2, && \text{for } 2 \le x \le 4 \\ &= 2ax + 5b, && \text{for } 4 < x \le 8 \end{aligned}$$

Find values of a and b.

iii) If $f(x) = \sqrt{x}$, $g(x) = \frac{1}{\sqrt{x}}$ in [a, b] in Cauchy's Mean value theorem. Show that C is a geometric mean between a and b.

iv) If $y = \sin(m\sin^{-1}x)$ prove that

a) $(1-x^2)\, y_2 - xy_1 - m^2y = 0$

b) $(1-x^2)\, y_{n+2} - (2n+1)xy_{n+1} - (n^2 - m^2)\, y_n = 0$

v) Show that

$$\int \frac{\sin 10x}{\sin x} dx = 2\left[\frac{\sin 9x}{9} + \frac{\sin 7x}{7} + \frac{\sin 5x}{5} + \frac{\sin 3x}{3} + \sin x\right]$$

Hence show that $\int_0^{\pi} \frac{\sin 10x}{\sin x} dx = 0$.

5. Attempt **any two** of the following. **12**

i) State and prove Cauchy's mean value theorem and using it find C for the function $f(x)$, $\sin x$, $g(x) = \cos x$ in $[0, \pi/2]$.

ii) State and prove Leibnitz's theorem.

iii) Prove that

$$\int \sin_x^m . \cos^n x\, dx \frac{(\sin x)^{m+1}.(\cos x)^{n-1}}{m+n} + \frac{n-1}{m+n} \int (\sin x)^m (\cos x)^{n-2} dx.$$

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