April 2015

Seat Number

* 0 0 9 6 *

MATHEMATICS PAPER - I (NEW) : MTH - 241
Complex Analysis
(24115)

P. Pages : 3

Time : Two Hours **Max. Marks : 40**

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicates full marks.

1. Attempt **any eight** of the following : **8**

a) Find the argument of $-1+i\sqrt{3}$.

b) Find the value of $\left|\frac{z-1}{1-\bar{z}}\right|$.

c) Evaluate $\lim_{z \to 0} \frac{\bar{z}}{z}$ along real axis.

d) State Cauchy Riemann, equations.

e) Define continuity of a function f(z) at $z = z_0$.

f) Using Cauchy's integral theorem evaluate $\int_{|z|=1} z^2\,dz$

g) State the series for $\frac{1}{1+z}$ for $|z|<1$

h) Find the poles of the function $f(z)=\dfrac{1}{z(z-1)^2}$.

i) Find the residue at simple pole for $f(z)=\dfrac{e^z}{z(z-1)^2}$.

j) If $z=e^{i\theta}$, then find the value of $\sin\theta$ in terms of z.

2. a) Attempt **any two** of the following : 6

i) For any two complex numbers z_1, z_2.
prove that $|z_1 \cdot z_2| = |z_1||z_2|$ and
$\arg(z_1 z_2) = \arg z_1 + \arg z_2$

ii) Compute the modulus and principle argument of $(-1+i)^3$.

iii) Simplify using DeMoivre's Theorem
$$\frac{(\cos 2\theta - i\sin 2\theta)^7 (\cos 3\theta - i\sin 3\theta)^{-5}}{(\cos 4\theta - i\sin 4\theta)^{12} (\cos 5\theta - i\sin 5\theta)^{-6}}$$

b) Find the sum of all cube roots of unity. 2

3. Attempt **any two** of the following : 8

a) Evaluate $\lim\limits_{z\to 1+i} \dfrac{z^4+4}{z-1-i}$

b) State and prove the necessary condition for f(z) to be analytic.

c) Find an analytic function $f(z)=u+iv$
whose immaginary part is $e^x(x\sin y + y\cos y)$.

4. a) Attempt **any two** of the following : 6

i) If f(z) be analytic in a region bounded by two simple closed curves C_1 and C_2 and also on C_1 and C_2 then prove that
$$\int_{C_1} f(z)\,dz = \int_{C_2} f(z)\,dz$$

ii) Verify Cauchy's integral theorem for $f(z) = z+1$ round the contour $|z|=1$.

iii) Evaluate $\int_{|z|=1} \frac{e^z}{z} dz$ and hence deduce that $\int_0^{2\pi} e^{\cos\theta} \cos(\sin\theta) d\theta = 2\pi$.

b) Expand $f(z) = \frac{1}{z-2}$ in Laurent's series valid for $|z| > 2$. 2

5. a) Find the residues of $f(z) = \frac{z^2}{(z-1)(z-2)}$ at its poles. 4

b) State and prove Cauchy's Residue Theorem. 4

OR

a) Compute the residue of $\frac{ze^z}{(z-1)^3}$ at its pole. 4

b) Evaluate by Cauchy's residue theorem $\int_C \frac{5z-2}{z(z-1)} dz$ where C is the circle $|z| = 2$ taken counter clockwise. 4

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