Nov-2015

Seat Number

| | | | | | |
|---|---|---|---|---|---|
| | | | | | |

*0166*

कणगा - 031 / 032

# MATHEMATICS PAPER - III : MTH - 113
## A) Geometry (111103)
## B) Discrete Mathematics (111104)

**P. Pages : 11**

*A) Geometry (111103)*

**Time : Two Hours** **Max. Marks : 60**

---

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to the right indicate full marks.

**1.** a) Attempt **any six.** 6

i) The two types of change of co-ordinate axis are ..... and .....
   a) Translation, Rotation b) Rectangular, Rotation
   c) Translation, Rectangular d) None of these

ii) The equation $7x^2 + 8xy + y^2 - 52x - 22y + 76 = 0$ represents.....
   a) Ellipse b) Hyperbola
   c) Parabola d) Circle

iii) The section of the sphere taken by the plane is.....
   a) Sphere b) Circle
   c) Plane d) None of these

iv) If the sphere $x^2+y^2+z^2+2yx+2vy+2wz+d=o$ passes through origin then....
   a) d=-1 b) d=0
   c) d=1 d) None of these

v) The general equation of the cone which passes through the co-ordinate axis is.....

a) $ax + by + cz = 0$ b) $fyz + gzx + gxy = 0$

c) $x^2 + y^2 + 2gx + 2fy = 0$ d) $fyz + gzx + hxy = 0$

vi) The constant angle between the ..... and the generator of a right circular cone is semi-vertical.

a) Axis b) Line

c) Normal d) None of these

vii) The d.r.s. of the generator of the right circular cylinder whose axis is $\frac{x}{2}=\frac{y}{3}=\frac{z}{5}$ are.....

a) 2,3,5 b) 1,2,3

c) 3,4,5 d) None of these

viii) In enveloping cylinder generator touches a given surface and are ..... to a given straight line.

a) Intersect b) Parallel

c) Perpendicular d) None of these

b) Attempt **any six.** 6

i) State whether true or false.
If the eccentricity e < l, the conic is ellipse.

ii) Fill in the blanks
To remove the xy term from the equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ the axes should be rotated through an angle $\theta$ given by tan2 $\theta$ =-----.

iii) State whether true or false
Two spheres with centres $c_1, c_2$ and radii $r_1$ and $r_2$ respectively cut orthogonally if $c_1 c_2 = r_1^2 + r_2^2$.

iv) Find the radius of the sphere $x^2+y^2+z^2-2y-2z=2$.

v) State whether true or false.
If a, b, c are the d. r. s. of any generator of the cone $f(x, y, z)=0$ with vertex origin then f(a, b, c)= 0.

vi) Define right circular cone.

vii) Define a cylinder.

viii) State whether true or false.
The guiding curve of a right circular cylinder is a circle.

**2.** Attempt **any six.** **12**

i) If the axes are rotated through an angle $\theta = \sin^{-1}3/5$ keeping the origin fixed then find the equations of rotation.

ii) Find the centre of the conic given by the equation $x^2 + 4xy + y^2 - 2x + 2y + 4 = 0$.

iii) Find the transformed form of the equation $x^2 + 4x + 3y - 5 = 0$. If the origin is shifted to the point $(-2, 2)$ without changing the axis.

iv) Find the equation of a sphere having $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ as the extremities of a diameter.

v) Find the equation of the sphere passing through (0, 0, 0), (a, 0, 0), (0, b, 0), (0, 0, c).

vi) State the condition that the general equation of second degree $ax^2 + by^2 + cz^2 + 2fyz + 2gxz + 2hxy + 2yx + 2vy + 2wz + d = 0$ should represent a cone.

vii) If the line $\frac{x}{2} = \frac{y}{-1} = \frac{z}{3}$ is the generator of the cone $x^2 + y^2 + z^2 + axy - xz = 0$ Find 'a'.

viii) Find the equation of the cylinder whose generators are parallel to the z-axis and intersect the curve $ax^2 + by^2 + cz^2 = 1$, $\ell x + my + nz = p$.

ix) Find the equation of the right circular cylinder of radius 2, whose axis passes through (1, 2, 3) and has d. r. s. 2, -3, 6.

**3.** Attempt **any four.** **12**

i) The axes are changed by chainging the origin to $(\alpha, 2)$. By this transformation the line given by $x + 2y + 3 = 0$ passes through the origin. Find the value of $\alpha$.

ii) If by change of axes, without change of origin, the expression $ax^2+2hxy+by^2$ becomes $a'x'^2+2h'x'y'+b'y'^2$ then prove that $a+b=a'+b'$.

iii) Find the radius of the circle $x^2+y^2+z^2-2y-4z=11, x+2y+2z=15$.

iv) Show that the spheres $x^2+y^2+z^2+4y-5=0$ and $x^2+y^2+z^2-6y+5=0$ touch each other externally.

v) Find the equation of the right circular cone with vertex at $(1, 2, -3)$, semi-vertical angle $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ and d. r. s. of axis as 1, 1, 0.

vi) Obtain the equation of cylinder whose generators are parallel to $\frac{x}{1}=\frac{y}{1}=\frac{z}{1}$ and whose guiding curve is $x^2+2y^2=1, z=3$.

4. Attempt **any three.** 12

i) Show that equation of a conic is a second degree equation in x and y.

ii) Change the origin to (1, 2) and transform $3x^2-10xy+3y^2+14x-2y+3=0$. Further rotate the axes through $\theta=\pi/4$ and find the final transform of the equation.

iii) Find the equation of the sphere which passes through the points $(2, 4, -1), (0, -4, 3), (-2, 0, 1)$ and $(6, 0, 9)$.

iv) Find the enveloping cone of the sphere $x^2+y^2+z^2-2x+4z-1=0$ with its vertex at (1, 1, 1)

v) Find the equation of the cylinder whose generators intersect the guiding curve $f(x,y,z)=0, ax+by+cz+d=0$ .

5. Attempt **any two.** 12

i) Find the equation of the tangent plane at any point $A(\alpha, \beta, \gamma)$ of the sphere $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ .

ii) Show that every homogeneous equation in x, y, z represent a cone with vertex at the origin.

iii) Find the equation of enveloping cylinder of the sphere $x^2+y^2+z^2-2x+3z+1=0$ whose generators are parallel to $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$.

**********

# B) Discrete Mathematics (111104)

**Time : Two Hours** **Max. Marks : 60**

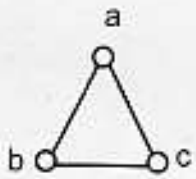
Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicates full marks.

**1.** a) Attempt **any six** of the following. **6**

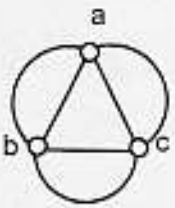
i) ..... is regular graph of degree 5.

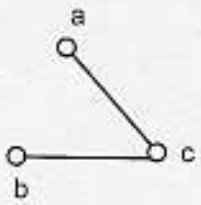
a) $K_4$ b) $K_5$
c) $K_6$ d) $K_7$

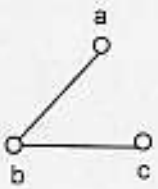
ii) Which graph is complement graph of 

?

a) 


b) 


c) 


d) 


iii) A graph $K_{m,n}$ is Hamiltonian if and only if.....

a) m>n b) M<n
c) m=n d) None of these

iv) Write the edge connectivity of $K_{3,5}$.
a) 3 b) 5
c) 8 d) 2

v) Chromatic number of graph $K_{3,2}$ is .....
a) 3 b) 5
c) 2 d) 1

vi) If G is connected plane graph with P vertices q edges and r faces then 2 = .....
a) p-q-r b) p-q+r
c) p+q-r d) p+q+r

vii) For any graph G with P vertices and q-edges then rank of G is.....
a) p-1 b) p+1
c) q+p-1 d) q-p+1

viii) If the graph $K_n$ is tree then n = .....
a) 2 b) 4
c) 6 d) 8

b) Attempt **any six** of the following. **6**

i) Find total number of edges of graph $K_{5,7}$.

ii) Define pseudograph.

iii) Define circuit in graph.

iv) Define Weighted graph.

v) Define colour class of graph.

vi) Define balanced diagraph.

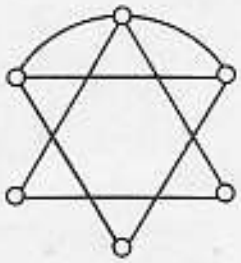
vii) What is branch node in tree ?

viii) Define spanning tree.

2. Attempt **any six** of the following. **12**

i) State Handshaking Lemma.

ii) Find the plane graph of graph.



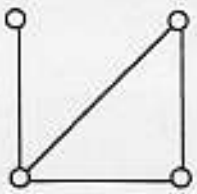
iii) Find the sum of two graphs $K_1$ and $K_3$.

iv) Determine the number of edges in a graph with 6 vertices, 2 of degree 4 and 4 of degree 2.

v) Construct a graph in which $K(G)<\lambda(G)<\delta(G)$.

vi) Define planar and plane graph.

vii) Draw geometrical dual of following graph G.



G

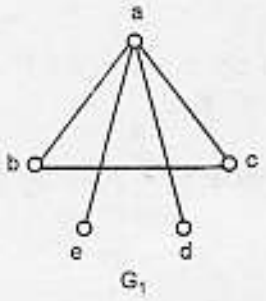
viii) Draw non isomorphic trees on six vertices.

ix) Find nullity of graphs $K_{3,3}$ and $N_6$.

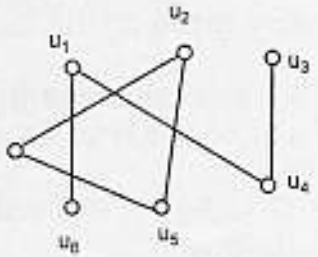
3. Attempt **any four** of the following. 12

i) Prove that the maximum number of edges in simple graph on n vertices is $\frac{n(n-1)}{2}$.

ii) Are the following graphs connected ? Justify.



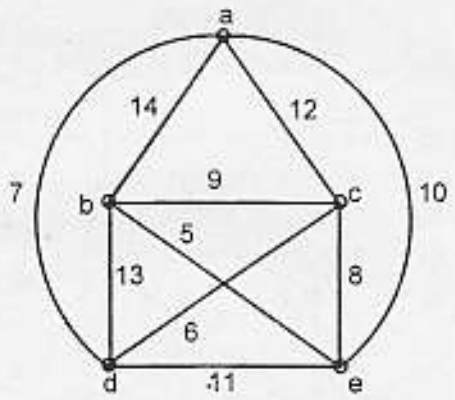

$G_1$




$G_2$

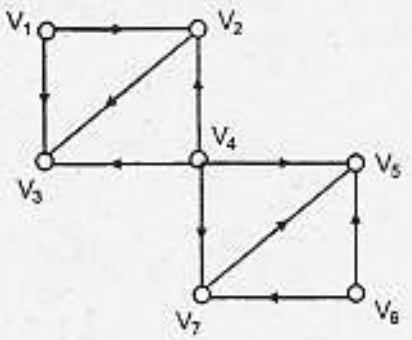
iii) Does there exists a graph on 5 vertices whose degree are 1,2,3,4 and 5 ? Justify.

iv) Using nearest neighbour method find the Hamiltonian cycle and its total distance for the graph given below.




v) Find the indegree and out degree of each vertex in the following diagraph and verify

$$\sum_{i=1}^{n} d^{-}(v_i) = \sum_{i=1}^{n} d^{+}(v_i).$$

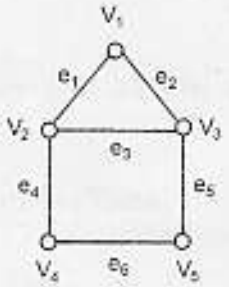


vi) Prove that there is unique path between every pair of vertices of a tree.

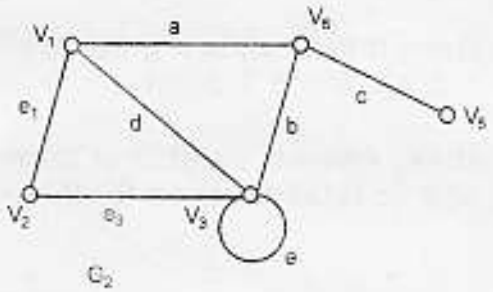
4. Attempt **any three** of the following. 12

i) If G is self complementary graph on n vertices then show that n is of the type 4K or 4K+1 for $K \in Z$.

ii) Find $G_1 \cup G_2$, $G_1 \cap G_2$ and $G_1 \oplus G_2$ where $G_1$ and $G_2$ are given below.



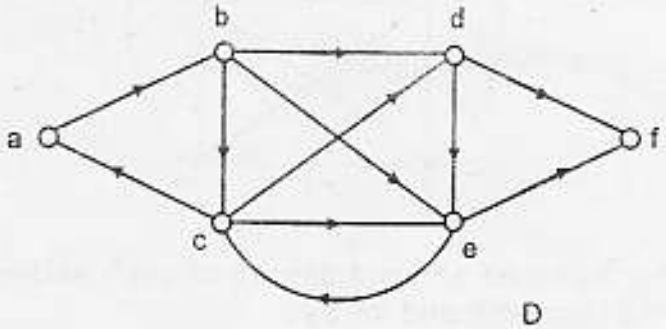

$G_1$




$G_2$

iii) Write short note on "Travelling Salesman Problem".

iv) In a diagraph D given below find all directed paths from a to f and directed circuits starting from d.

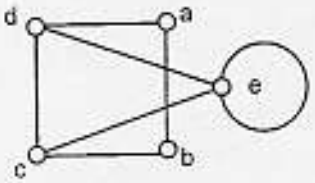



D

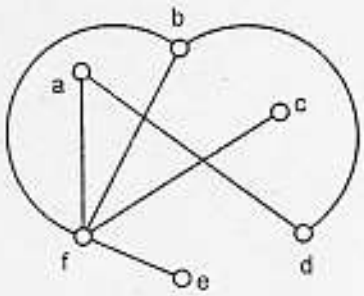
v) A tree has two vertices of degree 2 one vertex of degree 3 and three vertices of degree 4. How many number of vertices of degree 1 does it have ?

5. Attempt **any two** of the following. **12**

i) Are the following graphs are planar ? Justify.




$G_1$




$G_2$

ii) Define Eulerian circuit. Write short note on " Konigsberg seven bridges problem".

iii) Prove that in a binary tree with n vertices following.

a) The number of vertices is odd.

b) $P=\frac{(n+1)}{2}$, where P is number of pendant vertices.

c) q=p-1 where q is number of non pendant vertices.

*********