Seat Number

April - 2016

कडबा - 011

MATHEMATICS PAPER - I : MTH-231
Advanced Calculus
(23115)

P. Pages : 3

Time : Two Hours

Max. Marks : 40

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to the right indicate full marks.

1. Attempt **any eight** of the following. 8

a) Evaluate $\lim_{y \to o} \left\{ \lim_{x \to o} \frac{x^2 - y^2}{x^2 + y^2} \right\}$

b) Define continuity of a function f(x,y) at (a, b)

c) Write the condition for critical point (a, b) to be a saddle point.

d) State the formula for f_{xy} (a,b).

e) If $u = e^x \sin xy$, find $\frac{\partial u}{\partial y}$

f) Evaluate $\int_0^1 \int_0^1 xy \, dx \, dy$

g) Define absolute minimum of f(x, y) at (a, b)

h) Define a differentiable function f(x, y) at (a, b)

i) State the formula to obtain a volume of a solid V by triple integration.

j) Define a limit of a function f(x, y) as $(x,y) \to (a, b)$.

2. a) Attempt **any two** of the following. 6

i) State the necessary and sufficient condition for differentiability of a function.

ii) Find approximate value of
$(3.9)^2 (2.05) + (2.05)^3$

iii) If $u = \log(\tan x + \tan y + \tan z)$ prove that
$\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z} = 2$.

b) If $u = x^3z + xy^2 - 2yz$ then find the value of $\frac{\partial u}{\partial z}$ at the point (1, 2, 3) 2

3. Attempt **any two** of the following. 8

a) If $u = G^{-1}\left\{x^n f\left(\frac{y}{x}\right)\right\}$ and $G'(u) \neq 0$ then prove that
$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n\frac{G(u)}{G'(u)}$ and hence obtain the value of $xU_x + yU_y$ where
$u = \sin^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$.

b) If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ show that
$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$.

c) Verify Euler's theorem for $f(x,y) = ax^2 + 2hxy + by^2$.

4. a) Attempt **any two** of the following. 6

i) Explain Lagrange's method of undetermined multipliers to obtain extreme value of function $u = f(x, y, z)$

ii) Prove that

$$\sin(x+y) = (x+y) - \frac{(x+y)^3}{3!} + \text{-----}.$$

iii) Find the least value of the function $f(x,y) = xy + \frac{50}{x} + \frac{50}{y}$.

b) Find the critical point of $f(x,y)$ if $f(x,y) = x^3y^2(1-x-y)$ **2**

5. a) Using double integration find the area of the circle $x^2 + y^2 = 1$. **4**

b) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dxdydz}{\sqrt{1-x^2-y^2-z^2}}$ **4**

OR

a) Evaluate $\iint y\,dx\,dy$ over the area bounded by $y = x^2$ and $x + y = 2$ **4**

b) Use double integration to find the area of the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$. **4**

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