Nov-2016

Seat Number

काठ - 026

MATHEMATICS PAPER - I : MTH - 231
Calculus and Several Variables
(231101)

P. Pages : 4

Time : Two Hours

Max. Marks : 60

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to the right indicate full marks.

1. a) Attempt **any six** of the following. **6**

i) $\lim_{x\to 0} \lim_{y\to 0} y \cdot \sin\frac{1}{x}$.

a) 0 b) 1
c) -1 d) None of these

ii) If $u = x(1-y)$, $v = xy$ then $\frac{\partial(u,v)}{\partial(x,y)} = \ldots\ldots$

a) xy b) x
c) y d) None of these

iii) If Z is a homogeneous function of degree 3 then $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} =$.

a) 3 Z b) 2
c) 1 d) None of these

iv) If $z = f(x,y); x = \phi(u,v),\ y = \psi(u,v)$ then Z is a composite function of......

a) u and v b) x and y
c) u and x d) None of these

v) If $f(x,y) = 2x^2 - y^2 + 3$ then f has extreme value at ....

a) (1, 1) b) (0, 0)
c) (1, 0) d) None of these

vi) Expansion of x-y+3 in powers of (x-1) and (y-1) is...........

a) 3+ (x-1) - (y-1)
b) (x-1) - (y-1)
c) 3+ (x-1)
d) None of these

vii) $\int_0^1 \int_0^1 y \, dx \, dy = \ldots\ldots$

a) $\frac{1}{2}$
b) -1
c) 1
d) None of these

viii) The volume of the ellipse is obtained by evaluating.........

a) Line integral
b) Double integral
c) Triple integral
d) None of these

b) Attempt **any six** of the following. 6

i) If fxy and fyx are continuous at (a, b) then state fxy (a, b) = fyx (a, b) holds good or not ?

ii) Define differentiable function f (x, y) at (a, b).

iii) If $z = f(x, y) = x^2 + y^2$, where $x = t^2 + 1, y = 2t$ then find $\frac{dz}{dt}$.

iv) When Eulers Theorem is applicable ?

v) Write the condition for critical point (a, b) to be a function f (x, y) maximum.

vi) If $u = e^x \sin xy$, find $\frac{\partial u}{\partial x}$.

vii) Is the curve $y^2(a-x) = x^2(a+x)$ is symmetrical about the axis of x?

viii) Define Triple integral.

2. Attempt **any six.** 12

i) Evaluate $\lim_{(x, y) \to (0,0)} \frac{x^2 y^7}{x^4 + y^{14}}$ along $x^2 = y^7$.

ii) If $u = \log(x^2 + y^2 + z^2)$ then find $x \frac{\partial^2 u}{\partial y \partial z}$.

iii) Discuss the continuity of the function at (0, 0) where

$$f(x,y) = \frac{\sin(x^2+y)}{x+y}, \quad \text{when } (x,y) \neq (0,0)$$
$$= 0 \quad , \quad \text{when } (x,y) = (0,0)$$

iv) If $u = f\left(e^{y-z}, e^{z-x}, e^{x-y}\right)$ then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.

v) If $u = G^{-1}\left\{x^n f\left(\frac{y}{x}\right)\right\}$ and $G^1(u) \neq 0$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n\frac{G(u)}{G^1(u)}$.

vi) Expand xy + 3y - 2 in powers of (x-1) and (y + 2).

vii) State necessary condition for an extremum.

viii) Change the order of integration $\int_0^a \int_y^a \frac{x}{x^2+y^2}\,dx\,dy$.

ix) Discuss about axes intersection for the curve $ay^2 = 4x^2(a-x)$.

3. Attempt **any four** of the following. 12

i) Test the differentiability of the function

$$f(x,y) = \sqrt{|xy|}, \quad (x,y) \neq (0,0)$$
$$= 0 \quad , \quad (x,y) = (0,0)$$

at origin.

ii) Using differentials find the approximate value of.
$(5 \cdot 12)^2 (6 \cdot 85) - 3(6 \cdot 85)$.

iii) Verify Eulers theorem for the function $f(x,y) = x^3 + y^3 - 3x^2y$.

iv) If $f(x,y) = x^3 - xy^2$, show that θ used in the MVT applied to the points (2, 1) and (4, 1) satisfies the equation.
$3\theta^2 + 6\theta - 4 = 0$.

v) Expand e^x siny in powers of x and y as for as terms of third degree.

vi) Using double integration find the area of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$.

4. Attempt **any three** of the following. **12**

i) State and prove Young's theorem for equality of fxy and fyx at (a, b).

ii) If $f(x,y)=\frac{x^3y}{x^2+y^2}$, when $x^2+y^2 \neq 0$

$f(0, 0) = 0$.

Then show that $fxy(0,0) \neq fyx(0,0)$.

iii) If w = f (u, v) is a differentiable function of u and v and $u=\phi(x,y), v=\psi(x,y)$ are differentiable functions of x and y then prove that the composite function $w=f[\phi(x,y), \psi(x,y)]$ is a differential function of x and y and its first partial derivatives are given by $\frac{\partial w}{\partial x}=\frac{\partial w}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial w}{\partial v}\cdot\frac{\partial v}{\partial x}$ and $\frac{\partial w}{\partial y}=\frac{\partial w}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial w}{\partial v}\cdot\frac{\partial v}{\partial y}$

iv) Find the minimum and maximum distance from the origin to the curve $5x^2+6xy+5y^2=8$.

v) Using triple integration, find the volume of the sphere of radius r.

5. Attempt **any two** of the following. **12**

i) If $u=+an^{-1}\left(\frac{\sqrt{x^2+y^2}}{x-u}\right)$, find the value of $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x \partial y}+y^2\frac{\partial^2 u}{\partial y^2}$.

ii) State and prove Taylor's theorem for the function f (x, y).

iii) Evaluate $\iint_R e^{-x2}\,dx\,dy$, where R is the region bounded by the lines y = 0, x = 1 and y = x.

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