Nov-2015

Seat Number

00086

कर्ता - 026

MATHEMATICS PAPER - I : MTH-231
Advanced Calculus
(23115)

P. Pages : 3

Time : Two Hours

Max. Marks : 40

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicates full marks.

1. Attempt **any eight** of the following. **8**

a) Define continuity of a function f(x, y) at (a, b).

b) Define homogenous function of x & y with degree n.

c) Write the condition of critical point (a, b) to become a function f(x, y) maximum.

d) Evaluate $\int_0^1 \int_0^1 xy\,dx\,dy$

e) If $u = e^x \sin xy$, find $\frac{\partial u}{\partial x}$.

f) Evaluate $\lim_{(x, y) \to (0, 0)} \frac{x^3 - y^3}{x^3 + y^3}$ along y – axis.

g) If z = x+y, x = 2t, y = 3t then find $\frac{dz}{dt}$

h) Fill in the blank

The area of the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$ is .... unit^2.

i) State the formula for fy(a, b).

j) Define differentiable function of x and y at (a, b)

2. a) Attempt **any two** of the following. 6

i) State Schwarz's theorem and Young's theorem for f(x, y).

ii) let $f(x, y) = xy\frac{x^2 - y^2}{x^2 + y^2}$, $(x, y) \neq (0, 0)$

$f(0, 0) = 0$

Prove that $fxy(0, 0) \neq fyx(0, 0)$

iii) Find approximate value of $\sqrt{(5.98)^2 + (8.01)^2}$

b) If $u = x^3z + xy^2 - 2yz$ then find the value of $\frac{\partial u}{\partial y}$ at the point (1, 2, 3) 2

3. Attempt **any two** of the following. 8

a) If f is a homogeneous function of x and y of degree n prove that $xu_x + yu_y = nu$

b) If $u = f(y^2 - z^2, z^2 - x^2, x^2 - y^2)$, prove that $\frac{1}{x}\frac{\partial u}{\partial x} + \frac{1}{y}\frac{\partial u}{\partial y} + \frac{1}{z}\frac{\partial u}{\partial z} = 0$.

c) Let $z = f(u, v)$ where $u = 2x - 3y$ and $v = x + 2y$ prove that $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 3\frac{\partial z}{\partial v} - \frac{\partial z}{\partial u}$

4. a) Attempt **any two** of the following. 6

i) State Tyalor's theorem for function of two variables obtain Maclaurin's expansion from it.

ii) Express $x^3 + y^3 + xy^2$ in powers of (x – 1) and (y – 2).

iii) Investigate the maximum and minimum values of
$f(x, y) = 3x^2y - 3x^2 - 3y^2 + y^3 + 2$

b) State the condition for critical point to be a saddle point. 2

5. a) Using double integral find the area of the circle $x^2 + y^2 = a^2$. 4

b) Evaluate $\iiint (x + y + z)\,dxdydz$ over the tetrahedron x = 0, y = 0, z = 0 and x + y + z = 1. 4

OR

a) Change the order of integration and hence evaluate it. 4

$$\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\, dx\, dy$$

b) Find the volume of the sphere of radius 5. 4

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