April 2014

Seat No.

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कण - 065 / 066

MATHEMATICS PAPER - II (NEW) : MTH - 242

A) Topics in Differential Equations (24116) OR / B) Differential Equations and Numerical Methods (24117)

P. Pages : 4

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A) Topics in Differential Equations (24116)

Time : Two Hours **Max. Marks : 40**

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Answersheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicate full marks.

1. Attempt **any eight** of the following. **8**

a) Define Wronskian of $y_1(x)$ and $y_2(x)$.

b) The solution set of $\frac{dx}{yz} = \frac{dy}{xz} = \frac{dz}{xy}$ is

i) $x^2 - y^2 = c_1$ and $x + y = c_2 z$.

ii) $x^2 - y^2 = c_1$ and $y^2 - z^2 = c_2$

iii) $x^2 + y^2 = c_1$ and $y^2 - z^2 = c_2$

iv) None of these.

c) What is necessary condition for integrability of the Pfaffian differential equation $Pdx + Qdy + Rdz = 0$.

d) Define Beta function.

e) Find the Wronskian of the function $y_1 = \sin x$ and $y_2 = \sin x - \cos x$.

f) Define simultaneous differential equation of first order.

g) Show that $3x^2 y\, dx + x^3 dy = 0$ is exact.

h) What is value of integral $\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}$?

i) Show that $(y+z)dx + dy + dz = 0$ is integrable.

j) Two non zero functions $f_1(x)$ and $f_2(x)$ of the differential equation are linearly dependent iff the their Wronskian is ------- $\forall\ x \in (a, b)$
i) zero ii) non zero iii) Non vanishing d) None of these.

2. a) Attempt **any two** of the following. 6

i) If $y_1(x)$ and $y_2(x)$ are any two solutions of $a_0(x)y'' + a_1(x)\cdot y' + a_2(x)\cdot y = 0$ then show that the linear combination $c_1 y_1(x) + c_2 y_2(x)$, where c_1, c_2 are constant is also solution of given equation.

ii) Show that sin 3x and cos 3x are solution of the differential equation $y'' + 9y = 0$ and these are linearly independent.

iii) Using method of variation of parameter solve $\frac{d^2y}{dx^2} + a^2y = \operatorname{cosec}(ax)$.

b) Show that x and xe^x are linearly independent on the x-axis. 2

3. Attempt **any two** of the following. 8

i) Solve : $\frac{dx}{y^2} = \frac{dy}{x^2} = \frac{dz}{x^2y^2z^2}$ ii) Solve : $\frac{dx}{xy} = \frac{dy}{y^2} = \frac{dz}{zxy - 2x^2}$

iii) Solve : $\frac{yz\,dx}{y-z} = \frac{zx\,dy}{z-x} = \frac{xy\,dz}{x-y}$

4. a) Attempt **any two** of the following. 6

i) Solve $(x^2 - yz)dx + (y^2 - zx)dy + (z^2 - xy)dz = 0$

ii) Solve $(yz + z^2)dx - xz\,dy + xy\,dz = 0$

iii) Solve $3x^2dx + 3y^2dy - (x^3 + y^3 + e^{22})dz = 0$

b) Show that the equation $(yz - x^3)dx + (zx - y^3)dy + (xy - z^3)dz = 0$ is exact. 2

5. a) i) Evaluate $\int_0^1 (x\log x)^3 dx$ 4

ii) Prove that $\Gamma(m)\cdot\Gamma(m+\frac{1}{2}) = \frac{\sqrt{\pi}}{2^{2m-1}}\cdot\Gamma(2m)$ 4

OR

a) i) Evaluate $\int_0^\infty \frac{x^5}{5^x}dx$ 4

ii) Evaluate $\int_a^b (x-a)^m\cdot(b-x)^n dx$. 4

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B) Differential Equations and Numerical Methods (24117)

Time : Two Hours **Max. Marks : 40**

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Answersheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicate full marks.
6. Use of calculator is allowed.

1. Attempt **any eight.** **8**

a) State Lipschitz's condition.

b) Find Wronskian of $y_1(n) = \sin 2x \;\; y_2(n) = \cos 2x$.

c) Solve $\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$

d) State Necessary and Sufficient condition for integrability of the Pfaffian differential equation $Pdx + Qdy + Rdz = 0$.

e) Check exactness of the differential equation $(yz + 2x)\, dx + (zx - 2z)\, dy + (xy - 2y)\, dz = 0$

f) Give the integrating factor of homogeneous equation $Pdx + Qdy + Rdz = 0$ where $Px + Qy + Rz \neq 0$.

g) State Picards formula for solving the differential equation $\frac{dy}{dx} = f(x, y)$.

h) Choose the correct option.

Two nonzero solutions $y_1(x)$ and $y_2(x)$ of the diff equation are linearly dependent iff their Wronskian is ---------- $\forall\, x \in [a \;\; b]$

i) Zero ii) Nonzero iii) one iv) two.

i) Fill in the blanks

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R} = \frac{2dx - 3dy + 4dz}{\dots\dots\dots}$$

j) Write the corrector formula for solving the differential equation $\frac{dy}{dx} = f(x, y)\; y(x_0) = y_0$ by Adam Bashforth method.

2. a) Attempt **any two.** 6

i) By an example show that a continuous function may not satisfy Lipschitz's condition.

ii) Show that $y_1 = \sin 3x$ and $y_2 = \cos 3x$ are linearly independent solutions of the equation $y'' + 9y = 0$.

iii) Using method of variation of parameters solve $y'' + a^2 y = \sec ax$.

b) Find Wronskian of $e^{ax}\cos bx$ and $e^{ax}\sin bx$. 2

3. Attempt **any two.** 8

i) Solve $\frac{dx}{zx} = \frac{dy}{-zy} = \frac{dz}{z^2 + y^2}$

ii) Solve $\frac{dx}{\cot x} = \frac{dy}{\cot y} = \frac{dz}{\cot z}$

iii) Solve $\frac{yz\,dx}{y-z} = \frac{zx\,dy}{z-x} = \frac{xy\,dz}{x-y}$.

4. a) Attempt **any two.** 6

i) Solve $zy\,dx = zx\,dy + y^2 dz$

ii) Solve $yz\,dx + 2zx\,dy - 3xy\,dz = 0$

iii) Solve $xz^2\,dx - z\,dy + y\,dz = 0$.

b) Verify the condition of integrability for
$z\,dx + z\,dy + z(x + y + \sin z)\,dz = 0$ 2

5. a) Explain modified Euler's method and obtain the formula
$$y_1^{(n+1)} = y_0 + \frac{h}{2}\left[f(x_0\, y_0) + f(x_1\, y_1)^{(n)}\right]$$ 4

b) Using fourth order Runge Kutta's method find
$y(0\cdot1)$ if $\frac{dy}{dx} = x + y^2$ and $y(0) = 1$. 4

OR

a) i) Employ Picard's method of obtain $y(0\cdot1)$ for the equation
$\frac{dy}{dx} = x^2 + y^2$ given $y(0) = 0$. 2

ii) Obtain the Taylor's series expansion for $y(x)$
where $\frac{dy}{dx} = x - y^2$ and $y(0) = 1$. 2

b) Given $\frac{dy}{dx} = x^2(1+y)$ $y(1) = 1$ $y(1\cdot1) = 1\cdot233$ $y(1\cdot2) = 1\cdot548$
$y(1\cdot3) = 1\cdot978$ evaluate $y(1\cdot4)$ by Milne's predictor corrector method. 4

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