Nov-2016

कटी - 039

Seat Number

* 0 4 3 5 *

MATHEMATICS PAPER - II : MTH - 122
Theory of Numbers and Equations
(111202)

P. Pages : 4

Time : Two Hours

Max. Marks : 60

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicates full marks.

1. a) Attempt **any six** of the following. 6

i) If p is prime and p/ab then

a) p/a b) p/a or p/b

c) p/b d) p/a and p/b

ii) The number $\sqrt{2}$ is ------

a) Rational b) Irrational

c) Not a real number d) None of them

iii) Let $g.c.d(a,b) = d$ then

a) d/a only b) d/a and d/b

c) d/b only d) None of them

iv) If $a \cong b(\text{mod} n)$ then

a) $a - b = n$

b) $a - b = kn$ for some integer k

c) $a + b = kn$, for some integer k

d) None of them

v) If p is prime and $p/a_1, a_2, \ldots\ldots a_k$ then

a) $p = a_k$ for some k b) p/a_k for some k

c) $p = 2$ d) None of them

vi) If a/1 then

a) $a = \pm 1$ b) $a = 0$

c) $a = 1$ d) None of them

vii) Let a/bc with g.c.d. (a, b) = 1 then

a) a/1 b) a/c

c) $a \not| c$ d) None of them

viii) $\phi(7) = ...$

a) 6 b) 5

c) 4 d) 2

b) Attempt **any six** of the following. 6

i) State Fermat's theorem.

ii) Define Euler Function.

iii) Define equivalence classes.

iv) If $z_4 = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}\}$ is the set of all residue classes modulo 4. Find $\overline{2} \times_4 \overline{3}$.

v) Let α, β, γ and δ be the roots of equation. $ax^4 + bx^3 + cx^2 + dx + e = 0$ Find $\Sigma\alpha$.

vi) Find $\phi(12)$

vii) Define Greatest Common divisor.

viii) Define relatively prime integers.

2. Attempt **any six** of the following. 12

i) Show that 1357 and 1166 are relatively prime.

ii) If p is prime and $x^2 \equiv y^2 \pmod{p}$, prove that either $p/x+y$ or $p/x-y$.

iii) Find the elements of Z'_{16}.

iv) Form an equation whose roots shall be reciprocal of the root of equation $x^5 - 4x^3 + 6x^2 - 3x + 2 = 0$.

v) If a/b and b/a show that $a = \pm b$.

vi) To remove the second term from the equation $x^3 - 12x^2 + 48x - 72 = 0$, the roots are diminished by -------.

vii) State Descartes Rule of signs for positive root of $f(x) = 0$.

viii) Find the equation whose roots are negative of the roots of equation $x^7 + 4x^5 - 8x^3 + 6x^2 - 11x + 13 = 0$

ix) Define relation R as aR_b iff $a - b$ is an even integer determine equivalence class 3.

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3. Attempt **any four** of the following.

i) Using Fermat's theorem, Find the remainder when 8^{103} is divided by 13.

ii) Show that $\sqrt{5}$ is not rational number.

iii) If $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$ for $a,b,c,d \in Z$ and $n \in N$ Prove that
   i) $a + c \equiv b + d \pmod{n}$
   ii) $ac \equiv bd \pmod{n}$

iv) Prepare the composition table for addition and multiplication of residue classes modulo 7.

v) A relation R defined in the set of integer Z by xR_y iff $7x - 3y$ is divisible by 4. Show that R is an equivalence relation.

vi) Show by Induction principle, ' $7^n + 2$ is divisible by 3'.

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4. Attempt **any three** of the following.

i) Solve the equation $x^3 - 3x^2 + 4 = 0$ if two of its roots are equal.

ii) For the cubic equation $x^3 + px^2 + qx + r = 0$. Find value of the symmetric function $\sum\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)$.

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iii) Find the condition that the cubic equation $x^3 + px^2 + qx + r = 0$ should have two roots α, β connected by the relation $\alpha\beta + 1 = 0$.

iv) Write down the relation between roots and coefficient of cubic equation $ax^3 + bx^2 + cx + d = 0$.

v) If α and β are the roots of $ax^2 + bx + c = 0$.

Find the value of i) $\alpha^2 + \beta^2$ ii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$.

5. Attempt **any two** of the following. 12

i) Find the equation whose roots are the roots of $x^5 + 4x^3 - x^2 + 11 = 0$ each diminished by 3.

ii) Remove the second term from the equation $x^4 - 8x^3 + x^2 - x + 3 = 0$.

iii) Remove the fractional coefficients from the equation $5x^3 - \frac{3}{2}x^2 - \frac{3}{4}x + 1 = 0$.

OR

Explain Carden method of solving the cubic equation.

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