April-2016

B) Computational Algebra (23117)

Time : Two Hours **Max. Marks : 40**

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicate full marks.
6. Use of calculator is not allowed.

1. Attempt **any eight.** 8

a) If ϕ is Euler totient function find $\phi(11)$.

b) In the group $(Z_6 +_6)$ find $0(\bar{4})$.

c) Define identity element in a group $(G *)$.

d) Define cyclic group.

e) Let $H=\{\bar{0},\bar{3},\bar{6},\bar{9}\}$ be subgroup of $(Z_{12} +_{12})$ then obtain the coset $H+\bar{4}$.

f) Choose the correct option Let $f:G \to G'$ be a group homomorphism then f is...... iff ker f = {e} where e is identity of G.

i) On to   ii) One-one

iii) One-one and on to   iv) None

g) Let G be group of all nonsingular matrices of order n 0 under multiplication & IR^* be the group of all nonzero reals under multiplication Define $f:G \to IR^*$ as f (A) = |A| then show that f is a group homomorphism.

h) Define minimum distance of encoding function.

i) Define a group code.

j) Let $w_1 = 110101$ $w_2 = 001010$ find $\delta(w_1 w_2)$.

2. Attempt **any four.** 8

i) Consider Z the set of all integers and $*$ is defined on Z as $a*b=a+b+1$ then show that $(z*)$ is a group.

ii) In a group $(G*)$ prove that

a) Identity element is unique.

b) Every $a \in G$ has unique inverse.

iii) In a group $(G*)$ prove that

a) $(a^{-1})^{-1} = a$ b) $(a*b)^{-1} = b^{-1}*a^{-1}$

iv) If G is a group such that $a^{-1} = a \, \forall a \in G$ then show that G is abelian.

v) In the group $(Z_{12} +_{12})$ compute

a) $(\bar{5})^{-3}$ b) $(\bar{9})^{-2}$

vi) Show that order of every element in a finite group is finite.

3. Attempt **any two.** 8

i) Prove that every subgroup of a cyclic group is cyclic.

ii) Let G be a finite group, H is subgroup of G then prove that O(H) divides O(G).

iii) Show that every proper subgroup of a group of order 77 is cyclic.

4. a) Attempt **any two.** 6

i) Let (R +) be the group of reals under addition and $(R^* \circ)$ be the group of positive reals under multiplication. If $f:R \to R^*$ is defined as $f(x)=e^x \; \forall x \in R$ then show that f is an isomorphism.

ii) Let $f:G \to G'$ be a group homomorphism then prove that Ker (f) is subgroup of G.

iii) Prove that every infinite cyclic group is isomorphic to (Z +).

b) Consider (R +) be group of reals under addition and $f:R \to R$ defined by f(x) = x + 2 then check whether f is homomorphism or not. 2

5. a) Prove that an (m, n) encoding function can detect K or fewer errors iff it's minimum distance is at least K+1. 4

b) Let $H=\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 1\\ 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ be a parity check matrix determine the (2 5) group code. 4

OR

a) Let e be the (3, 5) encoding function defined as e(000) = 00000 e(010) = 01101 e(001) = 11110 e(011) = 10011 e(100) = 01010 e(101) = 10100 e(110) = 00111 e(111) = 11001 and d is maximum likelihood decoding function associated with e. Determine the no. of errors that (e d) can correct. 4

b) Compute 4

i) $\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\\ 1 & 1 & 1\end{bmatrix} \oplus \begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\\ 0 & 0 & 0\end{bmatrix}$

ii) $\begin{bmatrix}1 & 0\\ 1 & 1\\ 0 & 1\end{bmatrix} * \begin{bmatrix}1 & 1 & 0\\ 0 & 1 & 1\end{bmatrix}$

****