April 2017
MTH-242(B)

B) Differential and Difference Equations (241103)

Time : Two Hours **Max. Marks : 60**

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to the right indicate full marks.

1. a) Attempt **any six** of the following. **6**

i) The Wronskian of functions $y_1 = e^{2x}$ and $y_2 = e^{3x}$ is ----------.

a) e^{2x} b) e^{3x}

c) e^{5x} d) None of these

ii) Which of the following is the solution of differential equation $\frac{d^2y}{dx^2} = -9y$.

a) e^{3x} b) e^{-3x}

c) Sin 3x d) None of these

iii) Every continuous function ---------- satisfy Lipschitz's condition.

a) May b) Must

c) May not d) None

iv) $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R} = \frac{Pdx + Qdy + Rdz}{P^2 + ----}$.

a) $P^2 + R^2$ b) $P^2 + Q^2$

c) $Q^2 + R^2$ d) None of these

v) The solution of $\frac{dx}{z} = \frac{dy}{0} = \frac{dz}{-x}$ is ----------.

a) $y = c_1$ and $x^2 + y^2 = c_2$ b) $y = c_1$ and $x^2 + z^2 = c_2$

c) $y = c_1$ and $x + y = c_2$ d) None of these

vi) If the equation Pdx + Qdy + Rdz = 0 is homogeneous equation then integrating factor of this equation is ---------- if $\rho = Px + Qy + Rz \neq 0$.

a) ρ b) ρ^2

c) $\frac{1}{\rho}$ d) None of these

vii) The order of difference equation $y_{x+4} - y_x = 2$ is ----------.

a) 1 b) 2

c) 4 d) None of these

viii) The value of $\frac{1}{\phi(E)} a^x$ is ---------- if $\phi(a) \neq 0$ and symbols have their usual meaning.

a) $\frac{1}{\phi(a)}$ b) $\frac{1}{\phi(a)} a^x$

c) $\phi(a) a^x$ d) None of these

b) Attempt **any six** of the following. 6

i) Define : Wronskian of functions $y_1(x), y_2(x)$ and $y_3(x)$.

ii) Define : Linear combination of solutions of second order linear differential equation.

iii) State Lipschitz's condition.

iv) What is the sufficient condition of integrability of an Pfaffian differential equation Pdx + Qdy + Rdz = 0 ?

v) Show that $(y^2 + Z^2 + x^2)dx - 2xy\,dy - 2xz\,dz = 0$ is integrable.

vi) Is the equation xdx + ydy + zdz = 0 exact? Why?

vii) Solve : $y_{x+2} - 7y_{x+1} + 12y_x = 0$.

viii) Define : Linear difference equation.

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2. Attempt **any six** of the following.

a) Show that sin 2x and cos 2x are linearly independent solutions.

b) Find Wronskian of functions e^{2x} and $e^{x}\sin x$.

c) Show that $f(x,y) = xy^2$ does not satisfies Lipschitz's condition on strip $|x| \le 1$ and $|y| \le \infty$.

d) Solve : $\frac{dx}{0} = \frac{dy}{-z} = \frac{dz}{y}$.

e) Solve : $\frac{dx}{y} = \frac{dy}{-x} = \frac{dz}{0}$.

f) Show that equation $(y + z)\,dx + dy + dz = 0$ is integrable.

g) Show that equation $(2x + yz)\,dx + (xz - 2z)\,dy + (xy - 2y)\,dz = 0$ is exact.

h) If m_1, m_2 ---------- m_n are n-roots of homogeneous difference equation of n^{th} order is $y_{x+n} + a_1 y_{x+n-1} + a_2 y_{x+n-2} + - - - - + a_n y_x = 0$ $a_n \neq 0$ and if these roots are reals then write down general solution of given equation.

i) Solve : $y_{x+2} - 4y_{x+1} + 3y_x = 0$.

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3. Attempt **any four** of the following.

a) By example show that a continuous function may not satisfy Lipschitz's condition on a rectangle.

b) Using Wronskian, show that x, x^2, x^3 are linearly independent.

c) Solve : $\frac{dx}{\tan x} = \frac{dy}{\tan y} = \frac{dz}{\tan z}$.

d) Solve : $\frac{dx}{y} = \frac{dy}{x} = \frac{dz}{xyz^2(x^2 - y^2)}$.

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e) Show that $yz\,dx - xz\,dy - y^2dz = 0$ is integrable and hence find its solution.

f) Solve : $y_{x+2} - 3y_{x+1} + 2y_x = 1$.

4. Attempt **any three** of the following. 12

a) Using method of variation of parameters Solve : $y'' = x - y$.

b) Prove that two solutions $y_1(x)$ and $y_2(x)$ of equation $a_0(x)y'' + a_1(x)y' + a_2(x)y = 0$, $a_2(x) \neq o \forall x \in (a,b)$ are linearly independent if and only if their Wronskian is nonzero at some point $x_o \in (a,b)$.

c) Solve : $\dfrac{dx}{x^2 - y^2 - z^2} = \dfrac{dy}{2xy} = \dfrac{dz}{2xz}$.

d) Solve : $(2x^2 + 2xy + 2xz^2 + 1)dx + dy + 2z\,dz = 0$.

e) solve : $u_{x+2} - 5u_{x+1} + 6u_x = 5^x$.

5. Attempt **any two** of the following. 12

a) Solve : $\dfrac{dx}{x(2y^4 - z^4)} = \dfrac{dy}{y(z^4 - 2x^4)} = \dfrac{dz}{z(x^4 - y^4)}$.

b) State and prove necessary condition for integrability of the Pfaffian differential equation Pdx + Qdy + Rdz = 0.

c) With usual notations prove the following.

i) $\dfrac{1}{(E-a)^n} a^x = \dfrac{x(x-1)(x-2)----(x+n-1)}{n!} a^{x-n}$.

ii) $\dfrac{1}{\phi(E)}[a^x f(x)] = a^x \dfrac{1}{\phi(aE)} f(x)$.

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