गंध - 011 / 012

B) Differential and Difference Equations (241103)

Time: Two Hours Max. Marks: 60

Instructions to Candidates:

- 1. Do not write anything on question paper except Seat No.
- 2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
- 3. Students should note, no supplement will be provided.
- 4. All questions are compulsory.
- Figures to the right indicate full marks.
- 1. Attempt any six of the following.

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- The Wronskian of functions $y_1 = e^{2x}$ and $y_2 = e^{3x}$ is -----.

b) e^{3x}

c) e^{5x}

- d) None of these
- ii) Which of the following is the solution of differential equation

$$\frac{d^2y}{dx^2} = -9y$$

a) e^{3x}

b) e^{-3x}

c) Sin 3x

- d) None of these
- Every continuous function ----- satisfy Lipschitz's condition.
 - a) May

Must

c) May not

None

iv)
$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R} = \frac{Pdx + Qdy + Rdz}{P^2 + - - -}.$$

a) $P^2 + R^2$

b) $P^2 + Q^2$

c) $Q^2 + R^2$

d) None of these

v) The solution of
$$\frac{dx}{z} = \frac{dy}{0} = \frac{dz}{-x}$$
 is ----

- a) $y = c_1$ and $x^2 + y^2 = c_2$ b) $y = c_1$ and $x^2 + z^2 = c_2$ c) $y = c_1$ and $x + y = c_2$ d) None of these

vi) If the equation Pdx + Qdy + Rdz = 0 is homogeneous equation then integrating factor of this equation is ----- if $\rho = Px + Qy + Rz \neq 0$.

a) p

b) ρ^2

c) $\frac{1}{\rho}$

d) None of these

vii) The order of difference equation $y_{x+4} - y_x = 2$ is -----

a) 1

b)

c) 4

d) None of these

viii) The value of $\frac{1}{\phi(E)}a^{x}$ is ----- if $\phi(a) \neq 0$ and symbols have

their

usual meaning.

a) $\frac{1}{\phi(a)}$

b) $\frac{1}{\phi(a)}a^{3}$

c) $\phi(a)a^{x}$

d) None of these

b) Attempt any six of the following.

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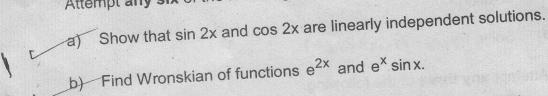
Define: Wronskian of functions $y_1(x), y_2(x)$ and $y_3(x)$

ii) Define: Linear combination of solutions of second order linear differential equation.

State Lipschitz's condition.

- iv) What is the sufficient condition of integrability of an Pfaffian differential equation Pdx + Qdy + Rdz = 0?
- v) Show that $(y^2 + Z^2 + x^2)dx 2xy dy 2xz dz = 0$ is integrable.
- vi) Is the equation xdx + ydy + zdz = 0 exact? Why?
- vii) Solve: $y_{x+2} 7y_{x+1} + 12y_x = 0$.
- viii) Define: Linear difference equation.

2. Attempt any six of the following.



c) Show that $f(x,y) = xy^2$ does not satisfies Lipschitz's condition on strip $|x| \le 1$ and $|y| \le \infty$.

d) Solve:
$$\frac{dx}{0} = \frac{dy}{-z} = \frac{dz}{y}$$
.

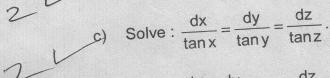
e) Solve:
$$\frac{dx}{y} = \frac{dy}{-x} = \frac{dz}{0}$$
.

- f) Show that equation (y + z) dx + dy + dz = 0 is integrable.
- g) Show that equation (2x + yz) dx + (xz 2z) dy + (xy 2y) dz = 0 is exact.
- h) If m_1 , m_2 ------ m_n are n-roots of homogeneous difference equation of n^{th} order is $y_{x+n} + a_1 y_{x+n-1} + a_2 y_{x+n-2} + --- + a_n y_x = 0$ $a_n \neq 0$ and if these roots are reals then write down general solution of given equation.
- i) Solve: $y_{x+2} 4y_{x+1} + 3y_x = 0$.

3. Attempt any four of the following.

a) By example show that a continuous function may not satisfy Lipschitz's condition on a rectangle.

b) Using Wronskian, show that x,x²,x³ are linearly independent.



d) Solve:
$$\frac{dx}{y} = \frac{dy}{x} = \frac{dz}{xyz^2(x^2 - y^2)}$$
.

- e) Show that $yzdx xzdy y^2dz = 0$ is integrable and hence find its solution.
- Solve: $y_{x+2} 3y_{x+1} + 2y_x = 1$.
- 4. Attempt any three of the following.

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- a) Using method of variation of parameters Solve : y'' = x y.
- Prove that two solutions $y_1(x)$ and $y_2(x)$ of equation $a_0(x)y'' + a_1(x)y' + a_2(x)y = 0$, $a_2(x) \neq o \ \forall x \in (a,b)$ are linearly independent if and only if their Wronskian is nonzero at some point $x_0 \in (a,b)$.

Solve:
$$\frac{dx}{x^2 - y^2 - z^2} = \frac{dy}{2xy} = \frac{dz}{2xz}$$

- d) Solve: $(2x^2 + 2xy + 2xz^2 + 1)dx + dy + 2zdz = 0$.
- solve: $u_{x+2} 5u_{x+1} + 6u_x = 5^x$.

Attempt any two of the following.

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a) Solve:
$$\frac{dx}{x(2y^4-z^4)} = \frac{dy}{y(z^4-2x^4)} = \frac{dz}{z(x^4-y^4)}$$
.

- b) State and prove necessary condition for integrability of the Pfaffian differential equation Pdx + Qdy + Rdz = 0.
- c) With usual notations prove the following.

i)
$$\frac{1}{(E-a)^n}a^x = \frac{x(x-1)(x-2)----(x+n-1)}{n!}a^{x-n}$$

ii)
$$\frac{1}{\phi(E)} \left[a^X f(x) \right] = a^X \frac{1}{\phi(aE)} f(x).$$
