113 (B) Numerical Methods
(111204)

Time : Two Hours **Max. Marks : 60**

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to the right indicate full marks.
6. Use of scientific calculator is allowed.

1. a) Attempt **any six** of the following. **6**

i) The root of the equation $x^3 - x - 4 = 0$ lies between

a) 0 and 1 b) 1 and 2

c) 2 and 3 d) 3 and 4

ii) $1 + \Delta = ...$

a) E^{-1} b) ∇

c) δ d) E

iii) If n values of f(x) are given, then $\Delta^n f(x)$ is ----

a) n b) 0

c) 1 d) n + 1

iv) One of the normal equations for fitting a straight line $y = a + bx$ is $\Sigma y_i = ...$

a) $na + b\Sigma x_i$ b) $na - b\Sigma x_i$

c) $a\Sigma x_i + b\Sigma x_i^2$ d) $na + b\Sigma x_i^2$

v) Better approximate value of the real root of the equation f(x) = 0 by Newton Raphson method is given by $x_{n+1} = ...$

a) $x_n - \frac{f'(x_n)}{f(x_n)}$ b) $x_n + \frac{f(x_n)}{f'(x_n)}$

c) $x_n - \frac{f(x_n)}{f'(x_n)}$ d) $x_n + \frac{f'(x_n)}{f(x_n)}$

vi) From the table

x	1	2	3	4
y	60	30	20	15

$\sum x_i^2$ is .....

a) 13 b) 30
c) 10 d) 125

vii) In Runge Kutta fourth order method $k_4 = ...$

a) $hf(x_0 + h, y_0 + k_2)$ b) $hf(x_0 + h, y_0 + k_3)$
c) $hf(x_0 + h, y_0 + k_1)$ d) $f(x_0 + h, y_0 + k_3)$

viii) In Euler's method $y_{n+1} = ...$

a) y_n b) $y_n + f(x_n, y_n)$
c) $y_n + hf(x_n, y_n)$ d) y_{n+1}

b) Attempt **any six** of the following. **6**

i) Define the central difference operator δ.

ii) Define the absolute error in r.

iii) $E^n y_k = ...$

iv) Runge Kutta Method of second order is the ----- method.

v) If the exact solution of the equation $y' = f(x, y)$ with $y(x_0) = y_0$, then Taylor's series expansion for y(x) about the point $x = x_0$ is $y(x) = ...$

vi) The iteration method is applicable to solve the equation $x = \phi(x)$ iff ....

vii) Prove that $\Delta = E\nabla$.

viii) The problem of fitting a power function $y = ax^b$ is nothing but the problem of fitting a ------ by ------ method.

2. Attempt **any six** of the following. **12**

i) Find the absolute error and relative error of the number 6.7 if both of its digits are correct.

ii) With usual notations, prove that

$$\mu = \frac{1}{2}\left[E^{1/2} - E^{-1/2}\right]$$

iii) State normal equations for fitting a second degree parabola $y = a + bx + cx^2$.

iv) Obtain Newton Raphson formula for square root of N.

v) State Runge Kutta fourth order formulae.

vi) Find the normal equations for fitting a curve $y = ae^{bx}$.

vii) Define the forward difference operator Δ and backward difference operator ∇.

viii) Find the first approximation of x for the equation $x = 0.21\sin(0.5 + x)$ by iteration method starting with $x_0 = 0.12$.

ix) Evaluate $\Delta^2(ab^{cx})$.

3. Attempt **any four** of the following. **12**

i) Find a real root of the equation $x^3 - 4x - 9 = 0$ by bisection method. Perform three iterations

ii) Show that

$$e^x\left(u_0 + x\Delta u_0 + \frac{x^2}{2!}\Delta^2 u_0 + \dots\right) = u_0 + u_1x + \frac{u_2x^2}{2!} + \dots$$

iii) Fit a straight line y = a + bx to the data

x	0	1	2	3
y	2	5	8	11

iv) If $y(1) = -3$, $y(3) = 9$, $y(4) = 30$ and $y(6) = 132$. Find four point Lagrange's interpolation polynomial that takes the same value.

v) Explain the iteration method for finding the real root of the equation $f(x) = 0$.

vi) Show that $\Delta(\log(f(x)) = \log\left(1 + \frac{\Delta f(x)}{f(x)}\right)$.

4. Attempt **any three** of the following. 12

i) Using Gauss's forward central difference formula, find the value of f (0.274) from the following table.

x	0.25	0.26	0.27	0.28	0.29	0.30
f(x)	4.000	3.846	3.704	3.571	3.448	3.333

ii) Given $\frac{dy}{dx} - 1 = xy$ and $y(0) = 1$. Obtain Taylor's series for y(x) and compute y(0.1) correct upto 4 decimal places.

iii) Given the set of values

x	10	15	20	25	30	35
y	19.97	21.51	22.47	23.52	24.65	25.89

From the difference table and write the values of $\Delta^2 y_{10}$, Δy_{20}, $\Delta^3 y_{15}$ and $\Delta^5 y_{10}$.

iv) Fit a second degree parabola to the following data.

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

v) Obtain the real root of the equation $x^3 - 2x - 5 = 0$ by Regula Falsi Method.

5. Attempt **any two** of the following. 12

i) Using Runge Kutta second order method, find y (0.2) and y (0.4). Given that y = 1 at x = 0 for h = 0.2 and $\frac{dy}{dx} = \frac{y-x}{y+x}$.

ii) State and prove Gauss's backward central difference formula.

iii) Explain Euler's modified method to solve the differential equation $\frac{dy}{dx} = f(x, y)$ with the initial condition $y(x_0) = y_0$.

****

116 / 4415