Nov-2016

Seat Number

* 0 5 8 2 *

काळ - 030 / 031

MATHEMATICS PAPER - II : MTH - 242

A) Topics in Differential Equations (24116) / B) Differential Equations and Numerical Methods (24117)

P. Pages : 4

A) Topics in Differential Equations (24116)

Time : Two Hours **Max. Marks : 40**

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicate full marks

1. Attempt **any eight** of the following. **8**

a) Define Linearly independent solution of a second order linear differential equation.

b) The solution set of $\frac{dx}{y} = \frac{dy}{-x} = \frac{dz}{o}$ is.

i) $x^2 + y^2 = c_1,\ y = c_2$ ii) $x^2 + y^2 = c_1,\ z = c_2$

iii) $x^2 - y^2 = c_1,\ z = c_2$ iv) None of these

c) State the necessary condition for integrability of the Pfaffian differential equation Pdx + Qdy + Rdz = 0.

d) What is the value of integral $\int_0^\infty e^{-x} \cdot x^6\, dx$?

e) What is the value of integral $\int_0^{\pi/2} \sin^p\theta \cdot \cos^q\theta\, d\theta$?

f) Show that $(y+z)dx+(z+x)dy+(x+y)dz=0$ is exact.

g) Find the Wornskian of cos 5x and sin 5x.

h) Find the multipliers of the differential equation.
$\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$.

i) Solve $(y+z)dx+dy+dz=0$.

j) Find the value of $\Gamma(5/2)$.

2. a) Attempt **any two** of the following. 6

i) If $y_1(x)$ and $y_2(x)$ are any two solutions of $a_0(x)y''(x)+a_1(x)y'(x)+a_2(x)y(x)=0$ then the linear combination $c_1y_1(x)+c_2y_2(x)$, where c_1 and c_2 are constant is also solution of given equation.

ii) Show that $y_1 = e^x \sin x$ and $y_2 = e^x \cos x$ linearly independent solution of the differential equation $y'' - 2y' + 2y = 0$.

iii) Solve by the method of variation of parameter $y'' + y - x = 0$.

b) Examine the function x^2, e^{2x}, e^{-2x} for the linear independence. 2

3. Attempt **any two** of the following. 8

i) Solve $\frac{dx}{z^2} = \frac{ydy}{xz^2} = \frac{dz}{xy}$.

ii) Solve $\frac{adx}{bc(y-z)} = \frac{bdy}{ca(z-x)} = \frac{cdz}{ab(x-y)}$.

iii) Solve $\frac{dx}{x+y} = \frac{dy}{x+y} = \frac{dz}{-x-y-2z}$.

4. a) Attempt **any two** of the following. 6

i) Solve $2yz\,dx + zx\,dy - xy(1+z)\,dx = 0$.

ii) Solve $(y^2+yz)dx + (z^2+zx)dy + (x^2-xy)dz = 0$.

iii) Solve $x\,dy - y\,dx - 2x^2 z\,dz = 0$.

b) Show that $(2x + y^2 + 2xz)dx + 2xy\,dy + x^2\,dz = 0$ is integrable. 2

5. a) 4

i) Evaluate $\int_0^\infty \frac{x^a}{a^x} dx$.

ii) Prove that $\beta(m, n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$. 4

OR

a) i) Prove that $\overline{|(n+1)} = m\,\overline{|n}$. 4

ii) Evaluate $\int_{-1}^{1} (1+x)^2 \cdot (1-x)^3\,dx$. 4

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