Seat Number

April 2015

* 0 4 8 2 *

MATHEMATICS PAPER - I : MTH - 231
Advanced Calculus (23115)

P. Pages : 3

Time : Two Hours **Max. Marks : 40**

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to the right indicate full marks.

1. Attempt **any eight** of the following. **8**

a) Define $\lim\limits_{(x,y)\to(a,b)} f(x,y) = \ell$

b) Evaluate $\lim\limits_{x\to 0}\left\{\lim\limits_{y\to 0}\frac{x^2-y^2}{x^2+y^2}\right\}$

c) State formula for fx (a, b)

d) If $u = f(x,y)$ and $x = \phi(t)$, $y = \psi(t)$ then state the chain rule for $\frac{du}{dt}$

e) Define homogenous function of two variables having degree n

f) Write the condition for critical point (a, b) to become a function f (x, y) minimum.

g) Evaluate $\int_1^2\int_0^1 (x^2+y^2)\,dx\,dy$

h) Area of ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ is.....unit² Fill in the blank.

i) Define absolute maximum of f(x, y) at (a, b).

j) Define saddle point.

2. a) Attempt **any two** of the following. 6

i) Let f(x, y) be a real valued function defined in a neighborhood of (a, b). If f is differentiable at (a, b) then prove that f is continuous at (a, b).

ii) Show that the function f where

$$f(x,y)=\frac{xy}{\sqrt{x^2+y^2}} \quad \text{if} \quad x^2+y^2\neq 0$$
$$=0 \quad \text{if} \quad x=y=0$$

Possesses partial derivatives but not differentiable at origin.

iii) Using differentials find approximate value of $\sqrt{(1\cdot02)^2+(1\cdot97)^3}$.

b) If $u=x^3z+xy^2-2yz$ then find the value of $\frac{\partial u}{\partial x}$ at (1, 2, 3). 2

3. Attempt **any two** of the following. 8

a) State and prove Euler's theorem for homogenous function of degree n in the two variables

b) If $u=f(e^{y-z},e^{z-x},e^{x-y})$ then prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$

c) If f (x, y) = x^2y+2xy^2, show that the value of θ in the expression of M.V.T. applied to the line segment joining the points (1, 2) to (3, 3) satisfies the equation $12\theta^2+30\theta-19=0$

4. a) Attempt **any two** of the following. **6**

i) State sufficient condition for a function f(x, y) to be extremum. at (a, b)

ii) Expand the function $f(x, y) = x^2+xy-y^2$ by Taylor's theorem in powers of (x-1) & (y+2)

iii) Find the stationary point and obtain the extreme value of $u = x^2 + y^2 + \frac{2}{x} + \frac{2}{y}$

b) State the simplified form of Maclaurin's expansion. **2**

5. a) Evaluate $\int\int_R xy(x+y)\,dx\,dy$ Where R is the area between $y=x^2$ & y=x. **4**

b) Evaluate $\int_0^3\left\{\int_0^2\left[\int_0^1 (x+y+z)\,dz\right]dx\right\}dy$ **4**

OR

a) Change the order of integration and hence evaluate **4**

$$\int_0^a\int_y^a \frac{x}{x^2+y^2}\,dx\,dy$$

b) Using triple integration find the volume of the sphere of radius a **4**

****