Nov - 2015

Seat Number

0160

MATHEMATICS PAPER - II : MTH- 242

(A) : Topics in Differential Equations (24116) / (B) : Differential Equations and Numerical Methods (24117)

P. Pages : 7

(A) : Topics in Differential Equations (24116)

Time : Two Hours **Max. Marks : 40**

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicate full marks.

1. Attempt **any eight** of the following. **8**

a) State Lipschitz condition for function f (x, y)

b) Find the wronskian of the functions e^x and xe^x.

c) The solution set of $\frac{dx}{x}=\frac{dy}{y}=\frac{dz}{z}$ is ....

i) $xy=c_1,\ yz=c_2$ ii) $x=c_1y,\ y=c_2z$

iii) $n=c_1z,\ y=c_2z$ iv) $xy=c_1z,\ y=c_2x$

d) Solve $\frac{dx}{zy}=\frac{dy}{zx}=\frac{dz}{xy}$.

e) Define pfaffian differential equation in n variables

f) Write the condition for the pfaffian differential equation $Pdx+Qdy+Rdz=0$ to be exact.

g) Define Gamma function.

h) Find value of $\beta(5,4)$.

i) The value of integral

$\int_0^1 x^2 (1-x)^{1/2}\, dx$ is .....

i) $\beta(3, \frac{1}{2})$ ii) $\beta(2, \frac{1}{2})$

iii) $\beta(3, \frac{3}{2})$ iv) $\beta(3, \frac{1}{3})$

j) Examine whether the set of functions $1+x, x^2, 1+2x$ are linearly dependent or not.

2. a) Attempt **any two** of the following. 6

i) If two solutions $y_1(x)$ and $y_2(x)$ of the equation

$a_0(x)\, y'' + a_1(x)\, y' + a_2(x)\, y = 0$

$a_0(x) \neq 0 \;\; \forall x \in (a,b)$

are linearly dependent then show that their Wronskian is identically zero.

ii) Show that $\sin 2x$ and $\cos 2x$ are solutions of the differential equation $y'' + 4y = 0$ and these are linearly independent.

iii) Solve $y'' + y = x$ by using method of variation.

b) Find the wronskian of $e^{ax} \cos bx$ and $e^{ax} \sin bx$, $b \neq 0$. 2

3. Attempt **any two** of the following. 8

i) Solve $\frac{dx}{\tan x} = \frac{dy}{\tan y} = \frac{dz}{\tan z}$

ii) Solve $\frac{dx}{x+z} = \frac{dy}{y} = \frac{dz}{z+y^2}$

iii) Solve $\frac{a\,dx}{bc(y-z)} = \frac{b\,dy}{ca(z-x)} = \frac{c\,dz}{ab(x-y)}$

4. a) Attempt **any two** of the following. **6**

i) Solve $(y+z)\,dx + dy + dz = 0$

ii) Solve $x\,dy - y\,dx - 2x^2 z\,dz = 0$

iii) Solve $\left(y^2 + z^2 - x^2\right)dx - 2xy\,dy - 2xz\,dz = 0$

b) Show that the equation $(yz+2x)dx + (zx-2z)dy + (xy-2y)\,dz = 0$ is exact. **2**

5. a) i) Prove that $\gamma(n+1) = \gamma(n)$. **4**

ii) Using β function **4**

Evaluate $\int_0^1 x^3\left(1-\sqrt{x}\right)^5 dx$

OR

i) Show that $\beta(m,n) = \frac{\gamma(m)\,\gamma(n)}{\gamma(m+n)}$ **4**

ii) Evaluate $\int_0^\infty x^8 e^{-x^3} dx$. **4**

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(B) : Differential Equations and Numerical Methods (24117)

Time : Two Hours **Max. Marks : 40**

Instructions to Candidates :

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2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicate full marks.

1. Attempt **any eight** of the following. **8**

a) Find Wronskian of e^{2x} and e^{3x}.

b) State the Lipschitz's condition.

c) Solve $\frac{dx}{x}=\frac{dy}{y}=\frac{dz}{z}$.

d) Fill in the blanks
$\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}=\frac{dx-dy+dz}{------}$

e) State the condition of exactness of pfaffian differential equation $Pdx+Qdy+Rdz=0$.

f) Show that $(2x+y^2+2xz)dx+2xydy+x^2dz=0$ is integrable.

g) Choose the correct option
Two solutions $y_1(x)$ and $y_2(x)$ of the equation
$a_0(x)y''+a_1(x)y'+a_2(x)y=0,\ a_0(x)\neq 0\ \forall x\in(a\ b)$ are Linearly dependent iff their wronskian is identically.
i) 1 ii) 0
iii) -1 iv) ∞

h) Give the Picards formula to obtain the solution of
$\frac{dy}{dx}=f(x,y)$.

i) Write the Taylor's series for $y(x)$ about $x = x_0$.

j) Choose the correct option.
According to Milnes predictor formula y_4 = ...

i) $y_0 + \frac{4h}{3}(2f_1 - f_2 + 2f_3)$

ii) $y_2 + \frac{h}{3}[f_2 + 4f_3 + f_4]$

iii) $y_0 + \frac{1}{6}[k_1 + 2(k_2 + k_3) + k_4]$

iv) None of these

2. a) Attempt **any two** of the following. 6

i) Prove that there exist two Linearly Independent solutions $y_1(x)$ and $y_2(x)$ of the equation
$a_0(x)y''(x) + a_1(x)y'(x) + a_2(x)y(x) = 0$
such that every solution $y(x)$ may be written as
$y(x) = c_1 y_1(x) + c_2 y_2(x) \forall x \in (a, b)$
where $c_1 \& c_2$ are suitable constants

ii) Show that $y_1 = e^x \sin x$ and $y_2 = e^x \cos x$ are Linearly Independent solutions of differential equation $y'' - 2y' + 2y = 0$.

iii) Using method of variation of parameters
Solve $\frac{d^2y}{dx^2} + a^2 y = \operatorname{cosec} ax$.

b) Find wronskian of $1, x, x^2$. 2

3. Attempt **any two** of the following. 8

i) Solve $\frac{dx}{\cot x} = \frac{dy}{\cot y} = \frac{dz}{\cot z}$

ii) Solve $\frac{dx}{x+y} = \frac{dy}{x+y} = \frac{dz}{-x-y-2z}$

iii) Solve by using method of multipliers

$$\frac{dx}{x(y^2+2)}=\frac{dy}{-y(x^2+2)}=\frac{dz}{z(x^2-y^2)}$$

4. a) Attempt **any two** of the following. 6

i) If the pfaffian differential equation $Pdx+Qdy+Rdz=0$ is integrable then show that

$$P\left(\frac{\partial Q}{\partial z}-\frac{\partial R}{\partial y}\right)+Q\left(\frac{\partial R}{\partial x}-\frac{\partial P}{\partial z}\right)+R\left(\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x}\right)=0$$

ii) Solve $2yz\,dx+2x\,dy-xy(1+2)dz=0$

iii) Solve $(y+2)dx+dy+dz=0$ by using auxiliary equations.

b) State when the equation $Pdx+Qdy+Rdz=0$ will be homogeneous. Check whether the following equation 2

$y(y+zx)dx+x(x-zy)dy+z(z-yx)dz=0$

is homogeneous or not.

5. a) Solve the differential equation $\frac{dy}{dx}=2x-y$ with $x_0=1, y_0=3$ by using Picard method of successive approximation. Find up to third approximation. 4

b) Using Runge kutta fourth order method find $y(0{\cdot}1)$ and $y(0{\cdot}2)$ given that 4

$\frac{dy}{dx}=xy+y^2$ $y(0)=1$

OR

a) By Milne's method find y(0.4) that satisfy the solution of 4
$\frac{dy}{dx}=xy^2+1$ with $y(0)=1, y(0.1)=1.105$ $y(0.2)=1.223, y(0.3)=1.354$.

b) Using Adams Bash forth predictor-corrector method find y (1.4) given that **4**

$y' = x^2 + x^2y$

$y(1) = 1,\ y(1.1) = 1.233\ y(1.2) = 1.548$

$y(1.3) = 1.979$

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