Nov-2015

Seat Number

0151

कणगा - 023

MATHEMATICS PAPER - II : MTH - 112
Calculus of One Variable (111102)

P. Pages : 4

Time : Two Hours

Max. Marks : 60

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to the right indicate full marks.

1. a) Attempt **any six** of the following : 6

i) $\lim_{x \to \infty} \frac{\log x}{x}$

a) 1 b) -1
c) 0 d) none of these

ii) $\lim_{x \to 0} \left[\frac{1}{x} - \frac{1}{\sin x}\right]$

a) 0 b) $\frac{1}{2}$
c) 1 d) none of these

iii) $f(x) = |x - a|$ then at $x = a$ $f(x)$ is
a) continuous b) discontinuous
c) differentiable d) none of these

iv) For which value of $c \in (0, 2\pi)$ the Rolle's theorem is verified for the function $f(x) = \sin x$ in $[0, 2\pi]$.
a) π b) $\pi/2$
c) $3\pi/2$ d) none of these

v) If $y = e^{3x}$ then $y_4 = \ldots\ldots$
a) y b) 27y
c) 81y d) none of these

vi) If $y = \sin(ax+b)$ then $y_n =$

a) $a^n.\sin\left(ax+b+\frac{n\pi}{2}\right)$ b) $a^n.\cos\left(ax+b+\frac{n\pi}{2}\right)$

c) $\cos(ax+b)$ d) none of these

vii) $1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\ldots$ is expansion of

a) e^x b) $\cos x$

c) $\sin x$ d) $\log(1+x)$

viii) $\int_0^{\pi/2} \cos^6 x\,dx =$

a) $\frac{5\pi}{32}$ b) $\frac{\pi}{32}$

c) $\frac{5}{32}$ d) none of these

b) Attempt **any six** of the following : 6

i) Define the continuity of the function at point x = a.

ii) Is rational function is continuous at every point in its domain?

iii) Define monotonic increasing function.

iv) Find the value of C using LMVT for the function $f(x) = x^2+1$, in [0,5].

v) n^{th} derivative of x^n is -----

vi) If $y = \cos(2x+5)$ then y_6 = ------

vii) Write the expansion of log (1 + x)

viii) Evaluate $\int_0^{\pi/2} \sin^{10} x\,dx$.

2. Attempt **any six** of the following : 12

i) Evaluate $\lim_{x\to 0}\left[\frac{1}{x}-\frac{1}{e^x-1}\right]$

ii) If $f(x)=\frac{2^x-1}{x}$, $x\neq 0$ is continuous at x = 0 then, find f(0).

iii) Discuss the continuity of $f(x) = \frac{\sin 3x}{x}$, for $x \neq 0$
$= 3$, for $x = 0$ at point $x = 0$

iv) Show that the function $f(x) = \cos x$ is decreasing function on $[0, \pi/2]$.

v) State Rolle's theorem.

vi) Find n^{th} derivative of $y = \cos^3 x$

vii) If $y = \log(x^2 + 3x + 2)$ find y_n.

viii) Write formula for $\int_0^{\pi/2} (\sin x)^m \cdot (\cos x)^n \, dx$

ix) Write the expansion of e^x.

3. Attempt **any four** of the following : 12

i) Examine the continuity of the function.

$$f(x) = \frac{x^2}{a} - a, \quad \text{for } 0 < x < a$$
$$= 0 \quad , \text{ for } x = a$$
$$= a - \frac{a^3}{x^2} \quad , \text{ for } x > a \text{ at point } x = a.$$

ii) If $f(x) = \frac{(3^{\sin x} - 1)^2}{\tan x . \log(1+x)}$ for $x \neq 0$ is continuous at $x = 0$ find $f(0)$.

iii) Discuss the applicability of Rolle's theorem for the function. $f(x) = \log\left[\frac{x^2 + ab}{x(a+b)}\right]$ in $[a, b]$, $0 \notin [a, b]$.

iv) Discuss the applicability of Cauchy's mean value theorem for the function $f(x) = e^x$, $g(x) = e^{-x}$ in $[a, b]$.

v) If $y = x^3 . \cos x$ find y_n.

vi) Evaluate $\int_0^{\infty} \frac{x^4}{(1+x^2)^5} dx$.

4. Attempt **any three** of the following : 12

i) Prove that every continuous function on closed and bounded interval is bounded.

ii) If $f(x) = x^2 + ax + b$, for $0 \le x < 2$
$= 4x - 1$, for $2 \le x \le 4$
$= ax^2 + 17b$, for $4 < x \le 6$
is continuous in [0, 6] find values of a and b.

iii) If $y = e^{m\sin^{-1}x}$ prove that
$(1-x^2)y_{n+2} - (2n+1)x\,y_{n+1} - (n^2+m^2)y_n = 0$

iv) Prove that $\frac{b-a}{\sqrt{1-a^2}} < \sin^{-1}b - \sin^{-1}a < \frac{b-a}{\sqrt{1-b^2}}$ if $0 < a < b < 1$
Hence show that $\frac{\pi}{6} - \frac{1}{2\sqrt{3}} < \sin^{-1}\frac{1}{4} < \frac{\pi}{6} - \frac{1}{\sqrt{15}}$.

v) Using Taylor's theorem expand $f(x) = 3x^3 - 2x^2 + x - 4$ in power of (x - 2).

5. Attempt **any two** of the following : 12

i) State and prove Cauchy's mean value theorem and find the value of C if $f(x) = x^2, g(x) = x^3$ in [1, 2].

ii) If $y = e^{ax}.\cos bx$ find y_n
and hence if $y = e^{3x}.\cos 5x$ find y_n.

iii) Find reduction formula for $\int \frac{\sin nx}{\sin x} dx$, $(n > 1)$
and show that $\int \frac{\sin 7x}{\sin x} dx = 2\left[\frac{\sin 6x}{6} + \frac{\sin 4x}{4} + \frac{\sin 2x}{2} + \frac{x}{2}\right]$.

****