oct -2014

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MATHEMATICS PAPER - I : MTH-111
Theory of Matrices
(11115)

P. Pages : 3

Time : Two Hours   Max. Marks : 40

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Answer sheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to the right indicates full marks.
6. Use of Calculator is not allowed.

1. Attempt **any eight** of the following. **8**

i) If $A=\begin{bmatrix} 2 & -3 \\ -2 & 5 \end{bmatrix}$ Find adj A.

ii) Fill in the blank with proper answer.
adj(A·B)=---------.

iii) Write down the elementary matrix $E_{23}(2)$ of order 3.

iv) If A and B are two matrices such that product AB is defined and $\rho(A)=2, \rho(B)=3$. What is $\rho(AB)$?

v) State the condition for consistency of non-homogeneous system of linear equation AX=B.

vi) Find the characteristic equation of the matrix $A=\begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$.

vii) Define orthogonal matrix.

viii) Write down the quadratic form of the matrix $A=\begin{bmatrix} 2 & 3 \\ 3 & 1 \end{bmatrix}$.

ix) If |A|=4, then A·adjA= ?

x) If λ is eigen value of non-singular matrix A, then what is the eigen value of A^{-1}.

2. a) Attempt **any two** of the following. **6**

i) For any square matrix A. Prove that A.adjA=adjA.A=|A|.I

ii) Let $A=\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ Find A^{-1} by using adjoint method.

iii) if $A=\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}, B=\begin{bmatrix} 5 & 0 \\ 1 & 2 \end{bmatrix}$ verify that $(AB)^{-1}=B^{-1}\cdot A^{-1}$.

b) If $A=\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ Find A_{13} and A_{23} **2**

3. Attempt **any two** of the following. **8**

i) If A is a matrix of rank r, prove that there exists non-singular matrices P and Q such that $PAQ=\begin{bmatrix} r & 0 \\ 0 & 0 \end{bmatrix}$.

ii) Determine the value of x that will make the matrix $A=\begin{bmatrix} x & x & 1 \\ 1 & x & x \\ x & 1 & x \end{bmatrix}$ of rank 2.

iii) Reduce the matrix A to its normal form and hence determine its rank where.

$$A=\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$$

4. a) Attempt **any two** of the following. **6**

i) State Cayley Hamilton theorem verify it for the matrix $A=\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

ii) Investigate for what values of λ and μ the system of equations.

$x+3y+2z=2$

$2x+7y-3z=-11$

$x+y+\lambda z=\mu$

have unique solution.

iii) Find Eigen values of the matrix $A=\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

b) Examine the system of equations for non-trivial solution 2

$$\begin{aligned} x+y+z &= 0 \\ 4x+y &= 0 \\ 2x+2y+3z &= 0 \end{aligned}$$

5. a)

i) Determine ℓ, m, n such that the matrix $A = \begin{bmatrix} 0 & 2m & n \\ \ell & m & -n \\ \ell & -m & n \end{bmatrix}$ is orthogonal and hence write down its inverse. 4

ii) Reduce the quadratic form 4

$x_1^2 + 2x_2^2 + 2x_3^2 + 2x_1x_2 - x_1x_3 + 2x_2x_3$

to its canonical form. Find its rank.

OR

i) Show that inverse of an orthogonal matrix is orthogonal, and show that 4

$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

is an orthogonal matrix.

ii) Express the quadratic form 4

$x_1^2 + 2x_2^2 - 7x_3^2 - 4x_1x_2 + 8x_1x_3$

in matrix notation. Find the rank of the quadratic form.

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