Oct-2014

Seat Number

* 0 2 9 6 *

कोकीळा - 023

MATHEMATICS PAPER - II : MTH - 112
Calculus
(11116)

P. Pages : 3

Time : Two Hours **Max. Marks : 40**

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Answer sheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to the right indicate full marks.
6. Use of calculator is not allowed.

1. Attempt **any eight** of the following. **8**

i) $\lim_{x \to 0} x^x$ is equal to

a) 1 b) -1
c) 2 d) none of these

ii) The function

$F(x) = x \sin \frac{1}{x}$ for $x \neq 0$

$F(0) = 0$

is ---------

a) Continuous and derivable
b) Not continuous but derivable
c) Continuous but not derivable
d) neither continuous nor derivable at the point x=0

iii) For which value of $C \in \left(0, \frac{\pi}{2}\right)$ the Rolle's theorem is applicable for the function F(x) = sinx + cosx in $\left[0, \frac{\pi}{2}\right]$.

iv) State Cauchy's Mean Value theorem.

v) For which value of x the function $F(x)=\frac{3x^2+4}{x-2}$ is discontinuous.

vi) Write the n^{th} derivative of e^{ax}.

vii) State Leibnitz's theorem for the n^{th} derivative of product of two functions.

viii) Write the Maclaurin's expansion of sinx.

ix) Evaluate $\int_0^{\pi/2}\cos^8 x\,dx$.

x) Evaluate $\int_0^{\pi/2}\sin^5 x\cos^4 x\,dx$.

2. a) Attempt **any two** of the following. **6**

i) If F(x) is continuous in [a, b] and $F(a)\neq F(b)$ then prove that F(x) assumes every value between F(a) and F(b).

ii) Evaluate $\lim_{x\to 4}\left(\frac{1}{\log(x-3)}-\frac{1}{x-4}\right)$.

iii) If

$$F(x)=\frac{\sin 4x}{5x}+a, \quad \text{if } x>0$$
$$=x+4-b, \quad \text{if } x<0$$
$$=1, \quad \text{if } x=0$$

is continuous at x=0 then show that $a+b=\frac{16}{5}$.

b) Evaluate $\lim_{x\to 0}\tan x\cdot\log x$. **2**

3. Attempt **any two** of the following. 8

i) State and prove Langrange's mean value theorem.

ii) Discuss the applicability of Rolle's theorem for the function $F(x)=(x-a)^m(x-b)^n$ in $[a, b]$, $m,n\in N$.

iii) Verify Cauchy's mean value theorem for the functions $F(x) = \sin x$, $g(x) = \cos x$ in $0\le x\le\frac{\pi}{2}$.

4. a) Attempt **any two** of the following. 6

i) Find the n^{th} derivative of $e^{ax}\sin(bx+c)$.

ii) If $y=x^2\log x$, find y_n.

iii) If $y=\sin\left(m\sin^{-1}x\right)$ then prove that $\left(1-x^2\right)y_{n+2}=(2n+1)xy_{n+1}+\left(n^2-m^2\right)y_n$.

b) Find n^{th} derivative of $\frac{1}{ax+b}$. 2

5. a) i) Obtain the reduction formula for $\int\frac{\sin nx}{\sin x}dx, (n>1), n\in N$. 4

ii) Prove that $e^x\cos x=1+x-\frac{x^3}{3}-\frac{x^4}{6}-\frac{x^6}{30}+\cdots\cdots$. 4

OR

i) Using Taylor's theorem, express the polynomial $2x^3+7x^2+x-6$ in the powers of (x-2). 4

ii) Evaluate $\int_0^\infty\frac{dx}{\left(a^2+x^2\right)^3}$. 4

****