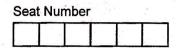
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## MATHEMATICS PAPER - II : MTH - 112 Calculus (11116)

P. Pages: 3

**Time: Two Hours** 

Max. Marks: 40

## Instructions to Candidates:

1. Do not write anything on question paper except Seat No.

- 2. Answer sheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
- 3. Students should note, no supplement will be provided.
- 4. All questions are compulsory.
- 5. Figures to the right indicate full marks.
- 6. Use of calculator is not allowed.
- 1. Attempt any eight of the following.

8

- i)  $\lim_{x\to 0} x^x$  is equal to
  - a) 1

b) -1

c) 2

d) none of these

ii) The function

$$F(x) = x \sin \frac{1}{x} \text{ for } x \neq 0$$

$$F(0)=0$$

is -----

- a) Continuous and derivable
- b) Not continuous but derivable
- c) Continuous but not derivable
- d) neither continuous nor derivable at the point x=0
- iii) For which value of  $C \in \left(0, \frac{\pi}{2}\right)$  the Rolle's theorem is applicable

for the function 
$$F(x) = \sin x + \cos x$$
 in  $\left[0, \frac{\pi}{2}\right]$ .

- iv) State Cauchy's Mean Value theorem.
- v) For which value of x the function  $F(x) = \frac{3x^2 + 4}{x 2}$  is discontinuous.
- vi) Write the nth derivative of eax.
- vii) State Leibnitz's theorem for the nth derivative of product of two functions.
- viii) Write the Maclaurin's expansion of sinx.
- ix) Evaluate  $\int_{0}^{\pi/2} \cos^8 x \, dx$ .
- x) Evaluate  $\int_{0}^{\pi/2} \sin^{5} x \cos^{4} x dx$ .
- 2. a) Attempt any two of the following.
  - If F(x) is continuous in [a, b] and F(a)≠F(b) then prove that F(x) assumes every value between F(a) and F(b).
  - ii) Evaluate  $\lim_{x\to 4} \left( \frac{1}{\log(x-3)} \frac{1}{x-4} \right)$ .
  - iii) If

$$F(x) = \frac{\sin 4x}{5x} + a, \quad \text{if } x > 0$$
  
= x + 4 - b , \quad \text{if } x < 0  
= 1 , \quad \text{if } x = 0

is continuous at x=0 then show that  $a+b=\frac{16}{5}$ .

6

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3. Attempt any two of the following.

8

- i) State and prove Langrange's mean value theorem.
- ii) Discuss the applicability of Rolle's theorem for the function  $F(x) = (x-a)^m (x-b)^n$  in [a, b],  $m,n \in \mathbb{N}$ .
- iii) Verify Cauchy's mean value theorem for the functions  $F(x) = \sin x, \ g(x) = \cos x \ \text{in} \ 0 \le x \le \frac{\pi}{2}.$
- 4. a) Attempt any two of the following.

6

- i) Find the  $n^{th}$  derivative of  $e^{ax} \sin(bx+c)$ .
- ii) If  $y = x^2 \log x$ , find  $y_n$ .
- iii) If  $y = \sin(m\sin^{-1}x)$  then prove that  $(1-x^2)y_{n+2} = (2n+1)xy_{n+1} + (n^2-m^2)y_n$ .
- b) Find n<sup>th</sup> derivative of  $\frac{1}{ax+b}$ .

2

5. a) i) Obtain the reduction formula for  $\int \frac{\sin nx}{\sin x} dx$ , (n > 1),  $n \in \mathbb{N}$ .

4

ii) Prove that  $e^x \cos x = 1 + x - \frac{x^3}{3} - \frac{x^4}{6} - \frac{x^6}{30} + \cdots$ 

4

OR

i) Using Taylor's theorem, express the polynomial  $2x^3 + 7x^2 + x - 6$  in the powers of (x-2).

4

ii) Evaluate  $\int_{0}^{\infty} \frac{dx}{(a^2 + x^2)^3}.$ 

4

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