Oct-2014

Seat Number

* 0 5 2 1 *

MATHEMATICS PAPER - I : MTH-231

Advanced Calculus

(New) (23115)

P. Pages : 3

Time : Two Hours **Max. Marks : 40**

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Answer sheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicates full marks.

1. Attempt **any eight** of the following. **8**

a) Show that repeated limits of $f(x,y)=\dfrac{x^2-y^2}{x^2+y^2}$ when $(x,y)\to(0,0)$ exist but unequal.

b) Define Differentiable function.

c) State Taylors theorem for a function of two variables.

d) Evaluate $\int_1^2 \int_0^x (x+2y)\,dy\,dx$

e) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2-y^2}{x^2+y^2}$ along $y=mx$ is ______.

f) If $u=\sin^{-1}\left(\dfrac{x^2+y^2}{x+y}\right)$ is a homogeneous function of degree ______.

g) If Z is a homogeneous function of degree 3 in (x,y) then

$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=$ ________.

i) 3 z ii) 2
iii) 1 iv) z

h) $f(a,b)$ is minimum if ________.

i) $\Delta>0$ and $r>0$ ii) $\Delta>0$ and $r<0$
iii) $\Delta<0$ iv) $\Delta=0$

i) Define Double Integral

j) Define Absolute and Relative Minimum.

2. a) Attempt **any two** of the following. 6

i) If $f(x, y)$ is defined in some neighborhood of (a, b) and f_x is continuous at (a, b) and f_y (a, b) exists then prove that f is differentiable at (a, b).

ii) Using differentials, find the approximate value of $(5.12)^2 (6.85) - 3(6.85)$

iii) Show that $f(x,y)=\frac{xy}{x^2+y^2}, x^2+y^2\neq 0$

$=0 \quad \text{if } x=y=0$

is not differentiable at (0, 0).

b) If u = f (y - z , z - x, x - y) then prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$ 2

3. Attempt **any two** of the following. 8

i) State and prove Euler's theorem for the homogeneous function f (x , y).

ii) If $z=\tan^{-1}(x/y)$ and $x=u+v,\ y=u-v$ show that $\frac{u-v}{u^2+v^2}$.

iii) If $u=\sin^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$ then prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}\tan u$.

4. a) Attempt **any two** of the following. **6**

i) State and Prove sufficient condition for the minimum value of the function f(x, y).

ii) Expand the function
$f(x,y)=x^2+xy-y^2$ by Taylors theorem in powers of
$(x-1)$ and $(y+2)$

iii) Find the minimum and maximum distance from the origin to the curve
$5x^2+6xy+5y^2=8$.

b) Explain working Rule to determine extreme values of the function. **2**

5. i) Evaluate **8**

$\iint_R xy(x+y)\,dx\,dy$ where R is the area between $y = x^2$ and $y = x$.

ii) Evaluate $\iiint \frac{dx\,dy\,dz}{(x+y+z+1)^3}$ over the region
$x \geq 0, y \geq 0, z \geq 0, x+y+z \leq 1$.

OR

i) Using triple integration, find the volume of the sphere of radius a

ii) Evaluate
$\iint xy\,dx\,dy$ over the rectangle bounded by x = 2, x = 5, y = 1, y = 2.

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