April - 2016

Seat Number

किनार - 028 / 029

MATHEMATICS PAPER - III : MTH-123 / 113
A) Laplace Transforms (111203) /
B) Numerical Methods (111204)

P. Pages : 8

123 (A) Laplace Transforms
(111203)

Time : Two Hours

Max. Marks : 60

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to the right indicate full marks.

1. a) Choose the correct option attempt **any six.** 6

i) $L\{\sinh 2t\} = ...$

a) $\frac{1}{s^2-4}$ b) $\frac{1}{s^2+4}$

c) $\frac{2}{s^2-4}$ d) $\frac{2}{s^2+4}$

ii) If $L\{F(t)\} = f(s)$ then $L\{F(at)\} = ..., a > 0$

a) $\frac{1}{a} f\left(\frac{s}{a}\right)$ b) $f\left(\frac{s}{a}\right)$

c) $f(as)$ d) $af(s)$

iii) $L^{-1}\left\{\frac{s}{s^2-9}\right\} = ..., s > 3$

a) $\frac{1}{3}\sin 3t$ b) $\cos 3t$

c) $\frac{1}{3}\sinh 3t$ d) $\cosh 3t$

iv) With usual notations, the value of $|*|*|$ is.....

a) $\frac{t^2}{2!}$ b) $\frac{t^3}{3!}$

c) $\frac{t^2}{3}$ d) $\frac{t^3}{3}$

v) $L^{-1}\left\{\frac{1}{(s+1)(s+2)}\right\} = ...$

a) $e^{-t} - e^{-2t}$ b) $e^{-2t} - e^{-t}$

c) $e^{t} - e^{2t}$ d) $e^{2t} - e^{t}$

vi) $L\{U(t-a)\} = ...$, where U (t – a) is Heaviside's unit step function.

a) e^{as} b) $\frac{e^{as}}{s}$

c) $\frac{e^{-as}}{s}$ d) e^{-as}

vii) $L^{-1}\left\{e^{-t}\,t\right\} = ...$

a) $\frac{1}{(s+1)^2}$ b) $\frac{1}{(s-1)^2}$

c) $\frac{1}{s^2}$ d) $\frac{1}{s+1}$

viii) $L\{\delta(t-2)\} = ...$

a) e^{2s} b) e^{-2s}

c) $\frac{e^{2s}}{s}$ d) $\frac{e^{-2s}}{s}$

b) Attempt **any six** of the following. 6

i) If $L\{F(t)\} = f(s)$ then state the formula for $L\left\{e^{at}F(t)\right\}$.

ii) Define sectionally continuous function on the interval [a, b]

iii) Find : $L^{-1}\left\{\frac{1}{4s^2+9}\right\}$.

iv) If $L^{-1}\{f(s)\} = F(t)$ and $F(0) = 0$ then what is $L^{-1}\{sf(s)\}$?

v) Define periodic function

vi) If $L^{-1}\{f(s)\} = F(t)$, $L^{-1}\{g(s)\} = G(t)$ then state the formula for $L^{-1}\{f(s)\,g(s)\}$.

vii) With usual notations, if $L\{F(t)\} = f(s)$ then what is $L\{F(t-a)\,U(t-a)\}$?

viii) Define Heaviside's unit step function.

2. Attempt **any six** of the following. 12

i) Find $L\{F(t)\}$, where $F(t) = \begin{cases} 5, & 0 < t < 3 \\ 0, & t > 3 \end{cases}$.

ii) Find $L\{2t^3 + 8\}$.

iii) Evaluate $L\{t \sin 2t\}$.

iv) Find $L^{-1}\left\{\frac{2s-5}{s^2-9}\right\}$.

v) If $L^{-1}\{f(s)\} = F(t)$ then show that $L^{-1}\{f^n(s)\} = (-1)^n t^n F(t)$.

vi) Draw the graph of the function

$$F(t) = \begin{cases} \sin t, & 0 < t < \pi \\ 0, & \pi < t < 2\pi \end{cases}$$

extended periodically with period 2π.

vii) Use convolution theorem to evaluate $L\left\{\frac{1}{s(s-1)}\right\}$.

viii) Show that $L\{t^2 U(t)\} = \frac{2}{s^3}$, with usual notations.

ix) Find $L\{t^2 \delta(t-4)\}$, with usual notations.

3. Attempt **any four** of the following. 12

i) Prove that $L\{\cosh kt\} = \frac{s}{s^2 - k^2}$, $s > |k|$

ii) Find $L\left\{\int_0^1 \sin 2u \, du\right\}$.

iii) Evaluate $L^{-1}\left\{\frac{4s+12}{s^2+8s+16}\right\}$

iv) Given that $L^{-1}\left\{\frac{s}{(s^2+1)^2}\right\}=\frac{1}{2}t\sin t$, find $L^{-1}\left\{\frac{1}{(s^2+1)^2}\right\}$.

v) Find $L^{-1}\left\{\frac{1}{(s+1)(s-2)}\right\}$ using convolution theorem.

vi) Solve : $y''(t)+y(t)=0$ subject to the conditions $y(0)=1, y'(0)=0$ using Laplace transform.

4. Attempt **any three** of the following. 12

i) Find $L\{F(t)\}$, if

$$F(t)=\begin{cases}(t-1)^2, & t>1\\ 0, & 0<t<1\end{cases}$$

ii) Show that $\int_0^{\infty}\frac{\sin t}{t}dt=\frac{\pi}{2}$.

iii) If $L^{-1}\{f(s)\}=F(t)$, then show that
$L^{-1}\{f(ks)\}=\frac{1}{k}F\left(\frac{t}{k}\right),\ k>0$.

iv) Evaluate $L^{-1}\left\{\frac{2s-1}{s^3-s}\right\}$

v) Find $L\left\{t^2U(t-4)\right\}$, where $U(t-4)$ is Heaviside's unit step function.

5. Attempt **any two** of the following. 12

i) Using Convolution Theorem, evaluate $L^{-1}\left\{\frac{1}{s^2(s+1)^2}\right\}$.

ii) Solve : $y''(t)+y(t)=t$ subject to $y(0)=1,\ y'(0)=-2$
using Laplace transforms.

iii) Prove that $L\{\delta(t-a)\}=e^{-as}$, where $\delta(t-a)$ is Dirac delta function.

****