April 2017

Seat Number

3 3 2 8 5 5

MATHEMATICS PAPER - I : MTH-231

Calculus and Several Variables

(231101)

P. Pages : 4

Time : Two Hours

Max. Marks : 60

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to the right indicate full marks.

1. a) Attempt **any six** of the following. 6

i) $\lim_{(x,y)\to(0,0)} \frac{x-a}{y-b}$ along the path y= 0 is ----------.

a) 0 b) $-\frac{a}{b}$

c) $\frac{a}{b}$ d) None of these.

ii) If $x=r\cos\theta$, $y=r\sin\theta$ then $\frac{\partial(x,y)}{\partial(r,\theta)}$ is -------------.

a) 1 b) 0

c) r d) None of these

iii) If $u=\sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$ ------------.

a) tan u b) cot u

c) sin u d) None of these

iv) If $u=f(x,y)$ and $x=\phi(t)$, $y=\psi(t)$ then u is a composite function of -----------.

a) x b) t

c) y d) None of these

v) If $f(x,y) = x^2 - 2y^2 + 1$ then f has extreme value at ---------.

a) (1, 1) b) (0,0)
c) (1,0) d) None of these

vi) Maclaurin's theorem for a function of two variables obtain from Taylors Theorem by putting--------.

a) a=0, b=0, h=x, k=y b) a=x, b=y, h=0, k=0
c) a=0, b=x, h=y, k=0 d) None of these

vii) $\int_0^1 \int_0^1 dx\, dy =$ -----------.

a) 1 b) $\frac{1}{2}$
c) -1 d) None of these

viii) The volume of the region V is obtained by evaluating -----------.

a) Triple integral b) Double integral
c) Line integral d) None of these

b) Attempt **any six** of the following. 6

i) When f xy(a,b) = f yx(a,b) holds good?

ii) Give formula for finding approximate value.

iii) If $u = f(x,y)$ and $x = \phi(t)$, $y = \psi(t)$ then the chain rule for $\frac{du}{dt}$ = ---------.

iv) What is the degree of homogeneous function $f(x,y) = \left(\frac{x^{\frac{1}{2}} + y^{\frac{1}{2}}}{x^{\frac{1}{3}} + y^{\frac{1}{3}}}\right)^{\frac{1}{2}}$

v) Write the condition for critical point (a, b) to be a saddle point.

vi) Write the remainder term in Taylor's theorem for a function of two variables after n terms.

vii) Is the point $(\frac{3a}{2}, \frac{3a}{2})$ lies on the curve $x^3 + y^3 = 3axy$?

viii) Define double integral.

2. Attempt **any six** of the following. 12

i) Evaluate $\lim_{(x,y) \to (0,0)} \frac{xy^3}{x^2 + y^6}$

ii) If $u = x\log(xy)$ then show that $\frac{\partial^2 u}{\partial x \partial y} = \frac{1}{y}$

iii) Discuss for continuity the function $f(x,y) = \frac{x^2 y}{x^4 + y^2}, (x,y) \neq (0,0)$

$f(0,0) = 0$ at $(0,0)$

iv) If $u = f(y-z, z-x, x-y)$ then show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$

v) State mean value theorem for function of two variables.

vi) Expand x y in powers of (x-1) and (y+2).

vii) State necessary condition for an extremum.

viii) Change the order of integration of $\int_0^a \int_x^a \frac{x}{\sqrt{x^2 + a^2}} dx\, dy$

ix) Discuss about axes intersection for the curve $x^2(x^2 + y^2) = a^2(y^2 - x^2)$

3. Attempt **any four** of the following. **12**

i) Show that the function $f(x,y) = \frac{xy}{\sqrt{x^2 + y^2}}$, if $x^2 + y^2 \neq 0$

$= 0$ if $x = y = 0$

is not differentiable at (0,0)

ii) Find approximate value of $\sqrt{(3.012)^2 + (3.997)^2}$.

iii) Verify Euler's theorem for $f(x,y) = ax^2 + 2hxy + by^2$

iv) If $f(x,y) = x^2 y + 2xy^2$, show that the value of θ in the expression of MVT applied to the line segment joining the points (1,2) to (3,3) satisfies the equation $12\theta^2 + 30\theta - 19 = 0$

v) Expand x^y at (1,1) up to terms of order 3.

vi) Find the area of the circle $x^2+y^2=25$.

4. Attempt **any three** of the following. **12**

i) State and prove necessary condition for differentiability.

ii) Let $f(x,y)=\dfrac{x^2-y^2}{x^2+y^2},(x,y)\neq(0,0)$. show that for the function f(x,y) both repeated limits exist but simultaneous limit does not exist.

iii) If W=f(u,v) is a differentiable function of u and v and $u=\phi(x,y)$, $v=\psi(x,y)$ are differentiable functions of x and y then prove that the composite function $W=f(\phi(x,y),\psi(x,y))$ is a differentiable function of x and y and its first partial derivatives are given by

$$\frac{\partial w}{\partial x}=\frac{\partial w}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial w}{\partial v}\cdot\frac{\partial v}{\partial x} \text{ and } \frac{\partial w}{\partial y}=\frac{\partial w}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial w}{\partial v}\cdot\frac{\partial v}{\partial y}.$$

iv) Divide 24 into three positive numbers such that their product is maximum.

v) Evaluate $\int_{x=0}^{1}\left\{\int_{y=0}^{2}\left[\int_{z=0}^{2} x^2yz\,dz\right]dy\right\}dx$

5. Attempt **any two** of the following. **12**

i) If $u=\sin^{-1}\sqrt{x^2+y^2}$ then prove that

$$x^2\frac{\partial^2u}{\partial x^2}+2xy\frac{\partial^2u}{\partial x\partial y}+y^2\frac{\partial^2u}{\partial y^2}=\tan^3u.$$

ii) State and prove Taylor's theorem for the function f(x, y).

iii) Evaluate $\iiint(x+y+z)\,dx\,dy\,dz$ over the tetrahedron $x=0,y=0,z=0$ and $x+y+z=1$

****