April 2015

Seat Number

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MATHEMATICS PAPER - II (NEW) : MTH - 242

(A) Topics in Differential Equations (24116) / (B) Differential Equations and Numerical Methods (24117)

P. Pages : 4

(A) Topics in Differential Equations (24116)

Time : Two Hours **Max. Marks : 40**

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicate full marks.

1. Attempt **any eight** of the following. **8**

a) State the Lipschitz condition.

b) Show that $y_1 = \sin 3x$ and $y_2 = \cos 3x$ are Linearly independent solution of differential Equation $y'' + 9y = 0$.

c) The solution set of $\frac{dx}{z} = \frac{dy}{o} = \frac{dz}{-x}$ is

i) $y = c_1$ and $x^2 + y^2 = c_2$ ii) $y = c_1$ and $x^2 + z^2 = c_2$

iii) $y = c_1$ and $x + y = c_2$ iv) None of these

d) Define Pfaffian differential equation in 3 variable.

e) What is the condition for exactness of the Pfaffian differential equation $Pdx + Qdy + Rdz = 0$.

f) Define Beta function.

g) $\Gamma(n+1) = -----$

h) Find Wronskian of e^x and xe^x.

i) Show that $y^2dx + 2xydy = 0$ is exact.

j) Define Lomogeneous equation.

2. a) Attempt **any two** of the following. **6**

i) Show that two solution $y_1(x)$ and $y_2(x)$ of the equation $a_0(x)y'' + a_1(x)y' + a_2(x)y = 0$, $a_0(x) \neq 0$, $\forall x \in (a,b)$ are linearly independent if and only if their Wronskian is non zero at some point $x_0 \in (a,b)$.

ii) Examine the existence and uniqueness of solution of the initial value problem $\frac{\partial y}{\partial x} = y^{1/3}$, $y(0) = 0$.

iii) Solve by the method of variation of parameter $x^2y'' + xy' - y = x^2e^x$.

b) Show that the function $1+x, x^2, 1+2x$ are linearly independent. **2**

3. Attempt **any two** of the following. **8**

i) Solve $\frac{dx}{\tan x} = \frac{dy}{\tan y} = \frac{dz}{\tan z}$.

ii) Solve $\frac{dx}{y} = \frac{dy}{x} = \frac{dz}{xyz^2\left(x^2 - y^2\right)}$.

iii) Solve $\frac{adx}{bc(y-z)} = \frac{bdy}{ca(z-x)} = \frac{cdz}{ab(x-y)}$.

4. a) Attempt **any two** of the following. **6**

i) Solve $xdy - ydx - 2x^2zdz = 0$.

ii) Solve $yzdx + 2zxdy - 3xydz = 0$.

iii) Solve $xz^2dx - zdy + ydz = 0$.

b) Solve $(a-z)(ydx + xdy) + xydz = 0$ **2**

5. a) i) Evaluate $\int_0^1 x^3 \cdot \log x \, dx$ **4**

ii) Prove that $B(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$ **4**

OR

a) i) Evaluate $\int_0^1 (x \log x)^4 dx$ **4**

ii) Show that $B(m,n) = \int_0^1 \frac{y^{m-1} + y^{n-1}}{(1+y)^{m+n}} dy$ **4**

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(B) Differential Equations and Numerical Methods (24117)

Time : Two Hours **Max. Marks : 40**

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicate full marks.
6. Use of calculator is allowed.

1. Attempt **any eight.** **8**

a) Define Wronskian of the functions $y_1(x)$, $y_2(x)$ and $y_3(x)$.

b) Show that e^{2x} and e^{3x} are L.I. on x-axis.

c) Choose the correct option.
Every continuous function ---------- satisfy a Lipschitz's condition on a rectangle.
i) may ii) must
iii) may not iv) None of these

d) Fill in the blanks $\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}=\frac{dx+2dy+3dz}{---------}$.

e) Solve dx=dy=dz.

f) Define Pfaffian differential equation in three independent variables x, y and z.

g) State the condition of exactness of the Pfaffian differential equation Pdx+Qdy+Rdz=0.

h) Show that the differential equation (y+z)dx+dy+dz=0 is integrable.

i) State Taylors series for y(x) at $x=x_0$ where $y(x_0)=y_0$.

j) Choose the correct option.
Fourth order Runge Kutta formula is
i) $y_{n+1}=y_n+\frac{1}{6}(k_1+k_2+k_3+k_4)$
ii) $y_{n+1}=y_o+\frac{1}{6}(k_1+2k_2+2k_3+k_4)$
iii) $y_{n+1}=y_n+\frac{1}{6}(k_1+2k_2+2k_3+k_4)$
iv) $y_{n+1}=y_o+\frac{1}{2}(k_1+2k_2+2k_3+k_4)$

2. a) Attempt **any two.** **6**

i) Check whether the function $f(x,y)=y^{3/4}$ defined on a rectangle $S=\{(x, y)/|x|\leq 1, |y|\leq 1\}$ satisfy Lipschitz's condition or not.

ii) If two solutions $y_1(x)$ and $y_2(x)$ of the equation $a_0(x)y''+a_1(x)y'+a_2(x)y=0$ $a_0(x)\neq 0$ are Linearly dependent then show that their Wronskian is zero.

iii) Using method of variation of parameters solve $y''-2y'+y=e^x$.

b) Find Wronskian of $e^x\cos x$ and $e^x\sin x$. 2

3. Attempt **any two.** 8

i) Solve $\frac{xdx}{y^2z}=\frac{dy}{zx}=\frac{dz}{yz}$.

ii) Solve $\frac{dx}{x+y}=\frac{dy}{x+y}=\frac{dz}{-x-y-2z}$.

iii) Solve $\frac{dx}{x(y^2+z)}=\frac{dy}{-y(x^2+z)}=\frac{dz}{z(x^2-y^2)}$.

4. a) Attempt **any two.** 6

i) Solve $2yzdx+zxdy-xy(1+z)dz=0$

ii) Solve $(y^2+z^2-x^2)dx-2xy\,dy-2xz\,dz=0$

iii) Solve $xz^2dx-zdy+2ydz=0$ by the method of auxiliary equation.

b) Show that the equation $(x^2-yz)dx+(y^2-zx)dy+(z^2-xy)dz=0$ is exact. 2

5. a) Find the value of y(0.1) given that $\frac{dy}{dx}=x+y^2$, $y(0)=1$ using Picards method. 4

b) If $\frac{dy}{dx}=xy+y^2$, $y(0)=1$ then find y(0.1) by using fourth order Runge Kutta method. 4

OR

a) If $\frac{dy}{dx}=-xy^2$ where y(z)=1 find y(2.2) by using Euler's modified method. Take h=0.1. 4

b) By using Adam-Bashforth method and Adam Moulten corrector formula obtain y(1.4) where $\frac{dy}{dx}=x^2y+x^2$ and y(1)=1, y(1.1)=1.233, y(1.2)=1.548, y(1.3)=1.979. 4

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