April 2014

Seat No.

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MATHEMATICS PAPER - I : MTH - 231

Advanced Calculus

(New) (23115)

P. Pages : 3

Time : Two Hours **Max. Marks : 40**

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Answersheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicates full marks.

1. Attempt **any eight** of the following. **8**

a) Evaluate $\lim\limits_{(x,y)\to(0,0)} \dfrac{\tan(x^2+y)}{x+y}$.

b) Find $\dfrac{dz}{dt}$ when $z = xy^2 + x^2y,\ x = at^2,\ y = 2at$.

c) State Maclaurins theorem for f (x, y)

d) Evaluate $\int_0^a \int_0^b (x^2+y^2)\,dxdy$.

e) $u = \tan^{-1} \dfrac{\sqrt{x^2+y^2}}{x-y}$ is homogeneous function of degree ....

f) If $f(x,y) = x^2 + xy - y^2$ then $f_{xy}(1,-2)$ = ...

g) Stationary points of the function u (x, y) are obtained by

i) $u_x = 0$ ii) $u_y = 0$

iii) $u_x = 0$ and $u_y = 0$ iv) None of these

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h) f(a, b) is maximum if

i) $\Delta > 0$ and $r > 0$

ii) $\Delta > 0$ and $r < 0$

iii) $\Delta < 0$

iv) $\Delta = 0$

i) Find critical point of the function $f(x, y) = 3x^2y - 3x^2 - 3y^2 + y^3 + 2$.

j) Define Absolute and Relative Maximum.

2. a) Attempt **any two** of the following. 6

i) Let f (x, y) be a real valued function defined on a neighbourhood of (a, b). If f is differentiable at (a, b) then

1) f is continuous at (a, b)

2) $f_x(a,b)$ and $f_y(a, b)$ exist.

ii) Using differentials, find approximate value of $(3.9)^2 (2.05) + (2.05)^3$.

iii) Let $f(x,y) = \dfrac{x^2 + y^2}{x^2 + y^2}$, $(x, y) \neq (0, 0)$ $f(0, 0) = 0$ then prove that

$f_{xy}(0, 0) = f_{yx}(0, 0)$.

b) Let $z = f(u, v)$ where $u = 2x - 3y$ and $v = x + 2y$ Prove that

$$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 3\frac{\partial z}{\partial v} - \frac{\partial z}{\partial u}.$$ 2

3. Attempt **any two** of the following. 8

i) State and prove mean value theorem for the function f(x,y).

ii) Verify Euler's theorem for the function $f(x,y) = x^3 + y^3 - 3x^2y$.

iii) If $u = \sin^{-1}\left[\dfrac{x^2 + 2xy}{\sqrt{x - y}}\right]^{\frac{1}{5}}$ then show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \dfrac{3}{10}\tan u$.

4. a) Attempt **any two** of the following. **6**

i) State and prove the necessary condition for extreme values.

ii) Expand $x^3 + y^3 + xy^2$ in powers of $(x-1)$ and $(y-2)$.

iii) Discuss the maxima and minima of the function $u = x^2 + y^2 + \frac{2}{x} + \frac{2}{y}$.

b) Write Lagranges method of undetermined multipliers. **2**

5. a) i) Using double integral find the area of the circle $x^2 + y^2 = a^2$. **8**

ii) Evaluate $\int_{y=0}^{3} \int_{x=0}^{2} \int_{z=0}^{1} (x+y+z)\, dz dx dy$

OR

i) Evaluate $\int_{0}^{a} \int_{x/a}^{x} \frac{x}{x^2 + y^2}\, dx dy$.

ii) Find the volume of the region bounded by the coordinate planes and the plane $x + y + z = 1$.

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