April 2014

Seat No.

कण - 143 / 144

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MATHEMATICS PAPER - II : MTH - 232

A) Topics in Algebra (New) (23116) OR / B) Computational Algebra (New) (23117)

P. Pages : 4

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A) Topics in Algebra (New) (23116)

Time : Two Hours **Max. Marks : 40**

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Answersheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to the right indicates full marks.

1. Attempt **any eight** of the following. **8**

a) Which is identity element in $\langle Z_8', x_8 \rangle$?

b) Let $G = \{1, -1, i, -i\}$ be a multiplicative group for $i^2 = -1$ find $0(-i)$.

c) If H is subgroup of finite group, G then $\frac{0(G)}{0(H)} = ?$

d) Prepare multiplicative composition table for Z_8'.

e) Define isomorphism of groups.

f) Define kerf.

g) Define Boolean ring.

h) Define integral domain.

i) State Euler's Theorem.

j) Find the generators of group $G = \{1, -i, -1, i\}$ w.r.t. multiplication of complex numbers.

2. a) Attempt **any two** of the following. **6**

i) Let G = {A : A is non - singular matrix of order n over IR} show that G is a group w.r.t. usual multiplication of matrices.

ii) Let G be a group and a, b, c $\in$ G prove that -
   i) $a \cdot b = a \cdot c \Rightarrow b = c$
   ii) $b \cdot a = c \cdot a \Rightarrow b = c$

iii) In $\langle Z_6, +_6 \rangle$ find
   i) $(\bar{2})^4$ ii) $(\bar{2})^6$ iii) $(\bar{2})^{-4}$

b) Show that $\langle Z_5, +_5 \rangle$ is cyclic group. **2**

3. Attempt **any two** of the following. **8**

a) Prove that non empty subset H of group G is subgroup of G iff
$a, b \in H \Rightarrow ab^{-1} \in H$

b) Show that every proper subgroup of group of order 65 is cyclic.

c) Find the remainder when 33^{19} is divided by 7.

4. a) Attempt **any two** of the following. **6**

i) Let $G = \{a^1, a^2, a^3, \ldots\ldots, a^{11}, a^{12} = e\}$ be a cyclic group of order 12 generated by a. Show that $f : G \to G$ defined by $f(x) = x^4 \ \forall\ x \in G$ is group homomorphism.

ii) Prove that every finite cyclic group of order n is isomorphic to $\langle Z_n, + \rangle$

iii) Let $f : G \to G'$ be a group homomorphism prove that kerf is subgroup of G.

b) Define automorphism of group. **2**

5. In ring $\langle Z_6, +_6, x_6 \rangle$
find :

i)	zero divisors in Z_6	ii)	multiplicative inverse of $\bar{4}$
iii)	additive inverse of $\bar{4}$	iv)	$(-\bar{7})\ x_6\ \bar{3}$
v)	$(-\bar{7})\ +_6\ \bar{3}$	vi)	unit element in Z_6 **8**

OR

i) Prove that every finite integral domain is a field. **4**

ii) Prove that ring IR is commutative iff
$(a+b)^2 = a^2 + 2ab + b^2$ 4

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B) Computational Algebra

(New) (23117)

Time : Two Hours **Max. Marks : 40**

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Answersheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to the right indicates full marks.
6. Use of calculator is not allowed.

1. Attempt **any eight** of the following. **8**

i) Define order of an element in a group.

ii) In the group $(Z_8 +_8)$ find $(\overline{3})^{-2}$

iii) Define cyclic group.

iv) Let $H = \{\overline{0}, \overline{2}, \overline{4}, \overline{6}\}$ be the subgroup of $(Z_8 +_8)$ then find the coset $H + \overline{3}$.

v) Define an Isomorphism.

vi) Consider IR^+ be the group of all +ve real numbers under multiplication and $(IR+)$ be the additive group of real numbers. If we define $\phi : IR^+ \to IR$ as $\phi(x) = \log(x)$ $\forall$ $x \in IR+$ then show that ϕ is a homomorphism.

vii) Let x = 1011101 and y = 0101010 then find $\delta(x, y)$.

viii) Define parity check code.

ix) If minimum distance of an encoding function is 3 then how many errors will e detect.

x) Fill in the blanks.
Every group of .... order is cyclic.

2. a) Attempt **any two** of the following. **6**

i) Show that the set of integers Z forms a group under the operation $a * b = a + b + 1$ $\forall$ $a, b \in Z$.

ii) Show that every proper subgroup of a group of order 77 is cyclic.

iii) Let G be a group and $a \in G, n \in Z$ then prove that $(a^n)^{-1} = (a^{-1})^n$.

b) Prove that every cyclic group is abelian. **2**

3. Attempt **any two** of the following. **8**

i) State and prove Lagranges theorem.

ii) Let H be the subgroup of a group G prove that

i) $Ha = Hb$ iff $ab^{-1} \in H$ ii) $a \in H$ iff $Ha=H$

iii) Using Fermats theorem find remainder when g^{87} is divided by 13.

4. a) Attempt **any two** of the following. **6**

i) Let $G = \{e, a, a^2, ...a^{11}\} a^{12} = e$ be a cyclic group of order 12. $f : G \to G$ is defined as $f(x) = x^4 \quad \forall\, x \in G$ then prove that f is a group homomorphism.

ii) Let $f : G \to G'$ be a group homomorphism then prove that kernel of f is a subgroup of G.

iii) Let G be a group and $g \in G$ be a fixed element the mapping $\phi : G \to G$ defined as $\phi(x) = gxg^{-1} \quad \forall\, x \in G$ then show that ϕ is an isomorphism.

b) Prove that the homomorphic image of an abelian group is abelian. **2**

5. a) Consider (2, 6) encoding function $e : B^2 \to B^6$ defined by e(00) = 000000 e(10)=101010 e(01) = 011110 e(11) = 111000
How many errors will e detect. **4**

b) Consider (2, 4) group code defined by e(00) = 0000 e(10) = 1000 e(01) = 0111 e(11) = 1111 decode the following words relative to maximum likelihood decoding function. **4**

i) 1010 ii) 0101 iii) 0111 iv) 1101

OR

a) Let $e : B^m \to B^n$ be a group code then prove that minimum distance of e is the minimum weight of non zero code word. **4**

b) Let $H = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$ be a parity check matrix decode the word 0101 relative to maximum likelihood decoding function. **4**

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