April 2014

Seat No. [ ]

* 0 3 9 0 *

MATHEMATICS PAPER - I : MTH - 241
Complex Analysis
(New) (24115)

P. Pages : 3

Time : Two Hours

Max. Marks : 40

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Answersheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicate full marks.

1. Attempt **any eight** of the following. 8

a) Find the sum of a complex number $z = x + iy$ and its conjugate $\bar{z}$.

b) Find amplitude of $-1+i$.

c) Evaluate $\lim_{z \to 0} \frac{\bar{z}}{z}$ along immaginary axis.

d) Define derivative of $f(z)$ at $z = z_0$.

e) Write the value of $f'(z)$ in terms of u_x and v_x where $f(z) = u + iv$.

f) State the series for e^z.

g) State Cauchy's integral formula for $f'(a)$.

h) Find the pole of $f(z) = \frac{1}{z^3(z+4)}$ which lies inside $|z| = 2$.

P.T.O.

कण - 022

i) Find the singular points of $f(z) = \frac{1}{z(z-i)}$

j) If $z = e^{i\theta}$ then find the value of $\cos\theta$ in terms of z.

2. a) Attempt **any two** of the following. 6

i) For any two complex numbers z_1, z_2 prove that $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$ and

$\arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2$

ii) If $\frac{z-1}{z+i}$ is purely immaginary, find the locus of z.

iii) Prove that $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ for $n \in N$.

b) Find cube roots of 1. 2

3. Attempt **any two** of the following. 8

a) Explain Milne - Thomson's method to obtain analytic function $f(z) = u + iv$ when u is given.

b) Show that the function $v(x, y) = e^{-y}\sin x$ is harmonic and find the analytic function f(z) = u + iv such that f(0) = 1.

c) If $f(z) = \frac{3z^4 - 2z^3 + 8z^2 - 2z + 5}{z - i}$ when $z \neq i$ $f(i) = 2 + 3i$
Examine f(z) for continuity at z = i.

4. a) Attempt **any two** of the following. **6**

i) State Cauchy Goursat theorem. Use it to evaluate $\int_{|z|=1} e^z\,dz$ and hence deduce that $\int_0^{2\pi} e^{\cos\theta} \cdot \sin(\theta + \sin\theta)\,d\theta = 0$

ii) Evaluate $\int_C \frac{3z-1}{z^2-2z-3}\,dz$ where C is the circle |z| = 4.

iii) Evaluate $\int_C \frac{dz}{z^3(z+4)}$ where C is the circle |z| = 2.

b) Expand $\frac{1}{z-2}$ in Taylor's series for |z| < 2. **2**

5. a) If f(z) is a complex variable function having simple pole at z = a and z = b is the pole of order m then state the formulae to obtain the residues at the poles a and b. **4**

b) Use contour integration to evaluate $\int_0^{2\pi} \frac{d\theta}{5+3\cos\theta}$ **4**

OR

a) Find the sum of residues of $\frac{e^z}{z^2+a^2}$ at its poles. **4**

b) Evaluate $\int_0^{\pi} \frac{2d\theta}{4+\sin^2\theta}$ **4**

****