Oct-2014

Seat Number

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MATHEMATICS PAPER - I : MTH-241
Complex Analysis
(New) (24115)

P. Pages : 3

Time : Two Hours

Max. Marks : 40

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Answer sheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicates full marks.

1. Attempt **any eight** of the following. 8

a) Express in the form x+iy if $z=\frac{1}{3+2i}$.

b) Find amplitude of -1-i.

c) Evaluate $\lim_{z\to i}\frac{z^5-i}{z+i}$.

d) Define Laplace differential equation.

e) Define analytic function.

f) Write the statement of Cauchy Goursat theorem.

g) State the series for $\frac{1}{1-z}$, for $|z|<1$.

h) Find the point of singularity of $f(z)=\frac{1}{z}$.

i) Find the residue of $f(z)=\frac{1}{z(z-1)^2}$ at simple pole.

j) If $z=e^{i\theta}$ then find the value of $\cos\theta$ in terms of z.

2. a) Attempt **any two** of the following. 6

i) For any two complex numbers z_1, z_2 prove that $|z_1+z_2| \le |z_1|+|z_2|$.

ii) If $|z_1|=|z_2|=|z_3|=5$ and $z_1+z_2+z_3=0$ then prove that $\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}=0$.

ii) Simplify using De Moivre's theorem $(\cos 3\theta + i\sin 3\theta)^8 (\cos 4\theta - i\sin 4\theta)^{-2}$.

b) Find cube roots of unity. 2

3. Attempt **any two** of the following. 8

a) Prove that if f(z) is differentiable at z_0 then it is continuous at z_0 but the converse is not true.

b) Show that the function $f(z)=\sqrt{|xy|}$ when $z \neq 0$ and f(0)=0 is not analytic at z=0 even though the C-R equations are satisfied at z=0.

c) Show that $\frac{1}{2}\log\left(x^2+y^2\right)$ satisfies Laplace equation. Find its harmonic conjugate.

4. a) Attempt **any two** of the following. 6

i) If f(z) is analytic in a simply connected region R then prove that $\int_a^b f(z)dz$ is independent of path of integration in R joining points a and b.

ii) Verify Cauchy's integral theorem for $f(z)=z^2$ round the circle $|z|=1$.

iii) Evaluate by Cauchy's integral formula $\int_C \frac{e^z}{z-2}$ where C is the circle $|z-2|=1$.

b) Evaluate $\int_C \frac{dz}{z}$ where C is the semicircular arc from -1 to 1 above the real axis. **2**

5. a) State and prove Cauchy's residue theorem. **4**

b) Evaluate $\int_C \frac{3z^2+2}{(z-1)(z^2+9)}dz$ by Cauchy's residue theorem where C is the circle $|z-2|=2$. **4**

OR

a) Evaluate $\int_{|z|=2} \frac{dz}{z^3(z+4)}$. **4**

b) Evaluate $\int_0^{2\pi} \frac{1}{(\cos\theta+2)^2}d\theta$. **4**

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